Acta Astronautica 68 (2011) 1379–1385

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Peristaltic transport of a viscoelastic fluid in a channel

Dharmendra Tripathi

Mathematics Group, Birla Institute of Technology and Science Pilani, Hyderabad Campus, Hyderabad 500078, Andhra Pradesh, India

A R T I C L E I N F O

Article history:
Received 25 June 2010
Received in revised form
13 August 2010
Accepted 15 September 2010
Available online 8 October 2010

Keywords:
Peristaltic transport
Fractional Maxwell model
Pressure difference
Friction force
Adomian decomposition method

A B S T R A C T

This paper is devoted to the study of the peristaltic transport of viscoelastic non-Newtonian fluids with fractional Maxwell model in a channel. Approximate analytical solutions have been constructed using Adomian decomposition method under the assumption of long wave boundary layer type approximation and low Reynolds number. The effects of relaxation time, fractional parameters and amplitude on the pressure difference and friction force along one wavelength are received and analyzed. The study is limited to one way coupling model with forward effect of the fluid on the peristaltic wall. It is evident from the result that pressure diminishes with increase in relaxation time and the effects of both fractional parameters on pressure are opposite to each other. The influences of these parameters on friction force are opposite to that of pressure.

1. Introduction

The propulsion of bio-fluids by continuous wavelike muscle contraction and relaxation of the walls of tubular organs such as oesophagus, intestine, ureter and blood vessels, is known as peristalsis. Latham [1] was probably the first to investigate the mechanism of peristalsis in relation to mechanical pumping. Since then, several investigators [2–5] have contributed to the study of peristaltic motion in both mechanical and physiological situations. In particular, Shapiro et al. [5] have investigated the peristaltic pumping under assumption of long wavelength and low Reynolds number. They considered the two-dimensional and axisymmetric flows of Newtonian fluid and they discussed the mechanical efficiency and some important phenomena of peristaltic pumps, such as reflux and trapping. Their investigation discussed only about Newtonian fluids and does not cover the peristaltic flow of other fluids, such as non-Newtonian fluids.

The non-Newtonian fluids are being considered more important and appropriate in view of engineering and biological applications as compared with the Newtonian fluids. Viscoelastic fluid is a non-Newtonian fluid, which contains both viscous and elastic properties. Most of the biological fluids such as blood, chyme, and food bolus are found to be viscoelastic in nature. Bohme and Friedrich [6] studied peristaltic flow of viscoelastic liquids. Some other workers [7–11] have investigated peristaltic transport of viscoelastic fluid with Maxwell model and they have discussed the effect of relaxation time on the peristaltic transport. Hayat et al. [12–15] have investigated the peristaltic transport of viscoelastic fluids with Jeffrey model and they have also discussed the effect of relaxation and retardation time on the peristaltic transport.

Shugan et al. [16] have studied about the two-way coupling model for wave motion in two-phase medium "fluid–elastic walls" accounting for interaction between those phases is considered within the frames of the boundary layer type approximation. Further, Shugan and Smirnov [17] have extended for peristaltic flow in a channel under standing wall's vibrations.

The constitutive equations with ordinary and fractional time derivatives have been introduced to describe the viscoelastic properties of materials in various fields. Rheological models with fractional time derivatives have played an important role in the study of the valuable tool of viscoelastic properties. In general, fractional Maxwell model

E-mail address: dtripathi.rs.apm@itbhu.ac.in

0094-5765/$ - see front matter
doi:10.1016/j.actaastro.2010.09.012

is derived from known Maxwell model by replacing ordinary derivatives of shear stress-strain relationship by derivatives of fractional order.

Recently, Hilfer [18] has developed a model of viscoelastic fluid with fractional derivative and shown its applications in physics. Using this fractional derivative model, some authors [19–21] have studied unsteady flow of viscoelastic fluids. They have obtained solutions by Laplace and Fourier transforms. Friedrich [22] has considered a particular case of this model to determine relaxation and retardation time with different fractional time derivatives in the shear stress-strain relationship by using the Riemann–Liouville definition. Taking Friedrich's model, some workers [23–27] have also investigated unsteady flow of viscoelastic fluids through channel and circular cylindrical tube. The solutions for velocity field and the associated shear stress have been obtained by using Laplace transform, Fourier transform, Weber transform, Hankel transform and discrete Laplace transform.

From the literature available, it is evident that no study on peristaltic transport of viscoelastic fluids with fractional Maxwell model under the assumption of long wavelength and low Reynolds number has been done. Adomian decomposition method (ADM) is used to obtain approximate analytical solution of the problem and the numerical results of the problem for different cases are depicted graphically. The effects of relaxation time, fractional parameters and amplitude on the pressure difference and friction force across one wavelength are discussed. The elegance of this method can be attributed to its simplistic approach in seeking the approximate analytical solution for the problem.

ADM was first proposed by Adomian [28–31] and used to solve a wide class of nonlinear and partial differential equations (PDE) [32–38]. ADM will be applied whenever it is appropriate to the solutions of PDE of any order. The numerical solutions reveal that, the method is user friendly, flexible, accurate, effective and powerful to solve large class of differential equations. ADM is newer approach to provide an approximate analytical solution to linear and nonlinear problems and it is particularly valuable as a tool for scientists, engineers and applied mathematicians. This technique provides immediate and visible symbolic terms of analytical solution, as well as numerical approximate solutions to both linear and nonlinear differential equations without linearization or discretization. The advantage of this method is its fast convergence of the solution.

2. Mathematical model

The constitutive equation of shear stress–strain relationship of viscoelastic fluid with fractional Maxwell model is given by

$$\left(1+\tilde{\lambda}_1^{\alpha}\frac{\partial^{\alpha}}{\partial\tilde{t}^{\alpha}}\right)\tilde{\tau}=G\tilde{\lambda}_1^{\beta}\frac{\partial^{\beta}\gamma}{\partial\tilde{t}^{\beta}}, \tag{1}$$

where, $\tilde{\lambda}_1$, $\tilde{t}$, $\tilde{\tau}$, γ are the relaxation time, time, shear stress, shear strain, $G=\mu/\tilde{\lambda}_1$ is the shear modulus, μ is the viscosity and α, β are the fractional parameters such that $0<\alpha\leq\beta\leq 1$. This model reduces to ordinary Maxwell model if $\alpha=\beta=1$ and Classical Navier Stokes model, when $\alpha=0$, $\beta=1$.

The governing equations of motion for incompressible fluids in two-dimensional case are

$$\left.\begin{aligned}\rho\left(\frac{\partial}{\partial\tilde{t}}+\tilde{u}\frac{\partial}{\partial\tilde{\xi}}+\tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)\tilde{u}=-\frac{\partial\tilde{p}}{\partial\tilde{\xi}}+\nabla\tilde{\tau},\\ \rho\left(\frac{\partial}{\partial\tilde{t}}+\tilde{u}\frac{\partial}{\partial\tilde{\xi}}+\tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)\tilde{v}=-\frac{\partial\tilde{p}}{\partial\tilde{\eta}}+\nabla\tilde{\tau},\end{aligned}\right\} \tag{2}$$

where, ρ, $\tilde{u}$, $\tilde{\xi}$, $\tilde{v}$, $\tilde{\eta}$, $\tilde{p}$are the fluid density, velocity, axial coordinate, transverse velocity, transverse coordinate and pressure.

The physical parameters are non-dimensionalized as follows:

$$\left.\begin{aligned}&\xi=\frac{\tilde{\xi}}{\lambda},\ \eta=\frac{\tilde{\eta}}{a},\ \lambda_1=\frac{c\tilde{\lambda}_1}{\lambda},\ t=\frac{c\tilde{t}}{\lambda},\ u=\frac{\tilde{u}}{c},\ v=\frac{\tilde{v}}{c\delta},\ h=\frac{\tilde{h}}{a},\\ &\phi=\frac{\tilde{\phi}}{a},\ \delta=\frac{a}{\lambda},\ p=\frac{\tilde{p}a^2}{\mu c\lambda},\ Q=\frac{\tilde{Q}}{ac},\ \tau=\frac{a\tilde{\tau}}{\mu c},\ \mathrm{Re}=\frac{\rho ca\delta}{\mu},\end{aligned}\right\} \tag{3}$$

where, $\tilde{h}$,$\tilde{\phi}$,$\tilde{Q}$ are transverse displacement of the walls, amplitude of the wave, volume flow rate and their counterparts without $\sim$ are the corresponding parameters in the dimensionless form. The parameters λ, a, c symbolize the wavelength, the semi-width of the channel and the wave velocity, respectively. Re stands for the Reynolds number while δ is defined as the wave number.

Using Eq. (1) in Eq. (2), in view of non-dimensionalisation and low Reynolds number approximation, reduced to

$$\left.\begin{aligned}&\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial p}{\partial\xi}=\lambda_1^{\beta-1}\frac{\partial^{\beta-1}}{\partial t^{\beta-1}}\left(\frac{\partial^2u}{\partial\eta^2}\right),\\ &\frac{\partial p}{\partial\eta}=0.\end{aligned}\right\} \tag{4}$$

Boundary conditions are given by

$$\left.\frac{\partial u(\xi,t)}{\partial\eta}\right|_{\eta=0}=0,\quad \left.u(\xi,t)\right|_{\eta=h}=0,\quad \left.\frac{\partial p}{\partial\xi}\right|_{t=0}=0. \tag{5}$$

Integrating Eq. (4) with respect to η, and using first condition of Eq. (5), we get

$$\frac{\partial^{\beta-1}}{\partial t^{\beta-1}}\left(\frac{\partial u}{\partial\eta}\right)=\lambda_1^{-(\beta-1)}\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial p}{\partial\xi}\eta. \tag{6}$$

Further integrating Eq. (5) from h to η, yields

$$\frac{\partial^{\beta-1}u}{\partial t^{\beta-1}}=\frac{\lambda_1^{-(\beta-1)}}{2}\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial p}{\partial\xi}(\eta^2-h^2). \tag{7}$$

The volume flow rate is defined as $Q=\int_0^h u\,d\eta$, which, by virtue of Eq. (7), reduces to

$$\frac{\partial^{\beta-1}Q}{\partial t^{\beta-1}}=-\frac{h^3\lambda_1^{-(\beta-1)}}{3}\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial p}{\partial\xi}. \tag{8}$$

The transformations between the wave and the laboratory frames, in the dimensionless form, are given by

$$x=\xi-t,\ y=\eta,\ U=u-1,\ V=v,\ q=Q-h, \tag{9}$$

where, the left side parameters are in the wave frame and the right side parameters are in the laboratory frame.