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The optimization of the orbital Hohmann transfer

Badaoui El Mabsout[a], Osman M. Kamel[b], Adel S. Soliman[c,*]

[a] *Laboratoire de Modélisation en Mécanique, Université Paris 6, Couloir 55-65, 4 Place Jussieu 75252 Paris cedex 05, Paris, France*
[b] *Astronomy and Space Sciences Department, Faculty of Sciences, Cairo University, Giza, Egypt*
[c] *Theoretical Physics Department, National Research Center, Dokki, Giza, Egypt*

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Abstract

There are four bi-impulsive distinct configurations for the generalized Hohmann orbit transfer. In this case the terminal orbits as well as the transfer orbit are elliptic and coplanar. The elements of the initial orbit a_1, e_1 and the semi-major axis a_2 of the terminal orbit are uniquely given quantities. For optimization procedure, minimization is relevant to the independent parameter e_T, the eccentricity of the transfer orbit. We are capable of the assignment of minimum rocket fuel expenditure by using ordinary calculus condition of minimization for $|\Delta V_A| + |\Delta V_B| = S$.

We exposed in detail the multi-steps of the optimization procedure. We constructed the variation table of $S(e_T)$ which proved that $S(e_T)$ is a decreasing function of e_T in the admissible interval $[e_{T\,\min}, e_{T\,\max}]$. Our analysis leads to the fact that $e_2 = 1$ for $e_T = e_{T\,\max}$, i.e. the final orbit is a parabolic trajectory.

Keywords: Orbital mechanics; Elliptic Hohmann orbital transfer; Optimization

1. Introduction

There are four distinct configurations for the generalized elliptic Hohmann orbit transfer (see [1,2]). Let us consider the first generalized Hohmann configuration where the apogee of the transfer orbit is equal to the apogee of the final orbit. Whence

$$a_1(1 - e_1) = a_T(1 - e_T); \quad a_2(1 + e_2) = a_T(1 + e_T),$$

where a_T and e_T are, respectively, the semi-major axis and the eccentricity of the transfer orbit.

Now if a_1, e_1 and a_2 are given, it is clear from the above first equation that a_T, e_T are dependent variables, so we can take one of these two parameters as the independent variable. Let e_T be this independent variable. If we take e_T as a parameter, we can calculate a_T from the first equality and $a_2(1 + e_2)$ from the second equality and we have

$$a_2(1 + e_2) = a_1(1 - e_1)(1 + e_T)/(1 - e_T).$$

Consequently a_T and $a_2(1 + e_2)$the distance from the focus to the apo–apse of the final orbit are uniquely calculated. Whence for a given e_T, we cannot choose this distance, but we have a family of final orbits.

Let us adopt the hypothesis that a_1, e_1 and a_2 are given, so e_2 is a function of e_T given by relation (12) written below. Consequently in the concerned optimization problem, we must consider that the minimization is

* Corresponding author.
E-mail addresses: bae@ccr.jussieu.fr (B. El Mabsout), adelssoliman7@yahoo.com (A.S. Soliman).

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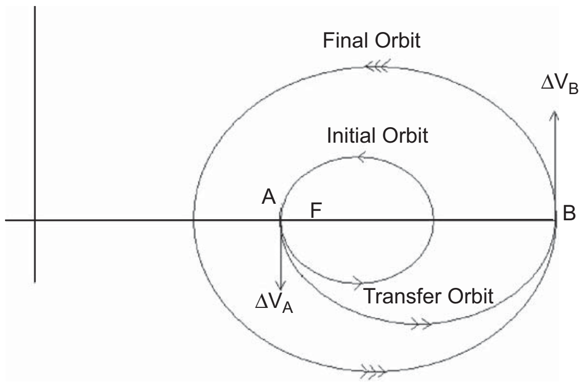

Fig. 1.

relevant to the independent parameter e_T. Moreover, we should minimize $|\Delta V_A + \Delta V_B| \equiv |\Delta V_A| + |\Delta V_B|$, not merely $(\Delta V_A + \Delta V_B)$ in order that we might minimize the consumption of rocket fuel. ΔV_A, ΔV_B are the increments of velocities at the peri–apse and the apo–apse of the initial elliptic orbit. The problem of orbit transfer is an old one. It turns back to 1925 when Engineer Hohmann introduced the problem in its simplest formulation (only two impulses and coplanar case) (see [3,4]. Extensive research work had been achieved since that time, and undoubtedly the problem became much more complicated, when considering the non-coplanar case and more than only two impulses, for instance the bi-elliptic, with plane change case (all trajectories are elliptic). We just mention here few investigators of this problem: Lawden, Prussing, Palmore, Roth, Marchal, Marec, and Barrar [5–11]. All of them investigated the optimization of the orbit transfer problem.

2. Method and results

We should minimize, for obvious reasons not the sum $(\Delta V_A + \Delta V_B)$, but the sum of the absolute values $(|\Delta V_A| + |\Delta V_B|) \equiv S$ (Fig. 1), where

$$V_A = \left\{\frac{\mu(1+e_1)}{a_1(1-e_1)}\right\}^{1/2} \text{ at point } A. \tag{1}$$

$$V_B = \left\{\frac{\mu(1-e_T)}{a_T(1+e_T)}\right\}^{1/2} \text{ at point } B. \tag{2}$$

$$\Delta V_A = -\left\{\frac{\mu(1+e_1)}{a_1(1-e_1)}\right\}^{1/2} + \varepsilon\left\{\frac{\mu(1+e_1)}{a_1(1-e_1)} + \frac{\mu}{a_1} - \frac{\mu}{a_T}\right\}^{1/2}; \quad \varepsilon = \pm 1. \tag{3}$$

with, $a_1 < a_T < a_2$

$$a_1 < a_T \Rightarrow \left\{\frac{\mu(1+e_1)}{a_1(1-e_1)} + \frac{\mu}{a_1} - \frac{\mu}{a_T}\right\}^{1/2} > \left\{\frac{\mu(1+e_1)}{a_1(1-e_1)}\right\}^{1/2}$$

so, we get $|\Delta V_A| = \{\mu(1+e_1)/a_1(1-e_1) + \mu/a_1 - \mu/a_T\}^{1/2} - \varepsilon\{\mu(1+e_1)/a_1(1-e_1)\}^{1/2}$. Then $\Delta V_A > 0$ if $\varepsilon = +1$, $\Delta V_A < 0$ if $\varepsilon = -1$, so we get

$$|\Delta V_A| = \varepsilon \Delta V_A = \left\{\frac{\mu(1+e_1)}{a_1(1-e_1)} + \frac{\mu}{a_1} - \frac{\mu}{a_T}\right\}^{1/2} - \varepsilon\left\{\frac{\mu(1+e_1)}{a_1(1-e_1)}\right\}^{1/2}. \tag{4}$$

For $|\Delta V_B|$ we have similarly the formula

$$|\Delta V_B| = \left\{\frac{\mu(1-e_T)}{a_T(1+e_T)} + \frac{\mu}{a_T} - \frac{\mu}{a_2}\right\}^{1/2} - \left\{\frac{\mu(1-e_T)}{a_T(1+e_T)}\right\}^{1/2}. \tag{5}$$

so, we have

$$S = |\Delta V_A| + |\Delta V_B|$$

$$= \left\{\frac{\mu(1+e_1)}{a_1(1-e_1)} + \frac{\mu}{a_1} - \frac{\mu}{a_T}\right\}^{1/2} - \varepsilon\left\{\frac{\mu(1+e_1)}{a_1(1-e_1)}\right\}^{1/2} + \left\{\frac{\mu(1-e_T)}{a_T(1+e_T)} + \frac{\mu}{a_T} - \frac{\mu}{a_2}\right\}^{1/2} - \varepsilon'\left\{\frac{\mu(1-e_T)}{a_T(1+e_T)}\right\}^{1/2}. \tag{6}$$

But ΔV_A should be < 0 or equivalently $\varepsilon = -1$, according to the orientation of the vector ΔV_A.

ΔV_B is positive (the vector ΔV_B is up oriented as $V_B \Rightarrow \varepsilon' = +1$ and we conclude that

$$S = \left\{\frac{\mu(1+e_1)}{a_1(1-e_1)} + \frac{\mu}{a_1} - \frac{\mu}{a_T}\right\}^{1/2} + \left\{\frac{\mu(1+e_1)}{a_1(1-e_1)}\right\}^{1/2} + \left\{\frac{\mu(1-e_T)}{a_T(1+e_T)} + \frac{\mu}{a_T} - \frac{\mu}{a_2}\right\}^{1/2} - \left\{\frac{\mu(1-e_T)}{a_T(1+e_T)}\right\}^{1/2}. \tag{7}$$

From Eqs. (4) and (5), we can easily derive the sum of

$$|\Delta V_A| = \sqrt{\frac{\mu}{b_1}}\{(1+e_1)^{1/2} + (1+e_T)^{1/2}\},$$

$$|\Delta V_B| = \sqrt{\frac{\mu}{b_2}}\{(1-e_2)^{1/2} - (1-e_T)^{1/2}\}, \tag{8}$$