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Square-root sigma-point Kalman filtering for spacecraft relative navigation

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ABSTRACT

A variant of sigma-point Kalman filters family called square-root unscented Kalman filter is derived to estimate the relative attitude and position of two spacecrafts referred to as the leader and follower. The square-root forms of unscented Kalman filter have a consistently increased numerical stability because all resulting covariance matrices are guaranteed to stay semi-positive definite. The general six degrees of freedom relative equations of motion are developed based upon the tensors. All leader states are assumed known, whereas the relative states are estimated using available line-of-sight observations between the vehicles along with acceleration and angular velocity measurements of the follower. The quaternion is used to describe the spacecraft relative attitude kinematics, while a three-dimensional generalized Rodrigues parameter is used to maintain the quaternion normalization constraint in the filter formulation. The simulation results indicate that the proposed filter can provide lower relative attitude and position estimation errors with faster convergence rates than the standard extended Kalman filter.

1. Introduction

Relative motion concepts between vehicles have been studied since the beginning of the manned space program. One such example is the spacecraft formations [1], which range from long baseline interferometry [2] to forming virtual structures [3], as well as rendezvous and docking maneuvers [4]. Another specific aircraft application is the autonomous aerial vehicles refueling in close proximity [5]. Many formation missions rely on accurate relative attitude and position knowledge between individual vehicles which are not generally available in practical applications. In order to obtain these quantities, it is necessary to develop high-precision relative navigation algorithms.

There have been some recent works dealing with relative navigation of vehicles. The relative navigation of a formation of uninhabited air vehicles (UAVs) is considered in [6]. The relative attitude and position estimation for spacecraft formations is examined in [7]. In the aforementioned two applications, multiple line-of-sight (LOS) observations are used to determine the relative attitude and position between vehicles. The LOS equipments, such as vision-based navigation (VisNav) system which mainly consists of an optical sensor and specific light sources (beacons), can be used to provide the LOS observations. The sensor is made up of a position sense diode (PSD) placed in the focal plane of a wide angle lens and an illustration of the VisNav sensor setup is provided in Fig. 1. The VisNav system has the advantage which is independent of any external systems and its detailed description can be found in [8,9].

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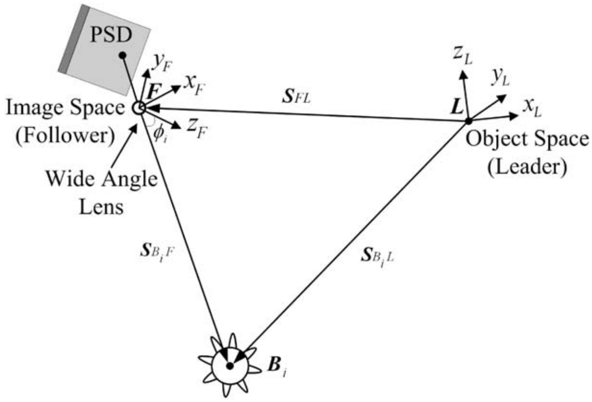

Fig. 1. Vision-based navigation system.

By far the most widely applied estimation algorithm for relative navigation has been the extended Kalman filter (EKF). However, poor performance or even divergence of the EKF has led to the development of other filters, such as the backwards-smoothing EKF [10]. Recently a variant of sigma-point Kalman filters (SPKFs) family called unscented Kalman filter (UKF) has been proposed for the integrated global positioning system (GPS) and inertial navigation [11]. Complete coverage of SPKFs is beyond the scope of this paper and we focus on UKF. The UKF, which generalizes elegantly to nonlinear systems without the burdensome analytic derivation or Jacobians as in the EKF, has two implementation variants, standard UKF [12] and square-root UKF (SR-UKF) [13,14]. The SR-UKF has consistently increased numerical stability comparing to the standard UKF because all resulting covariance matrices are guaranteed to stay semi-positive definite.

In this paper, the relative navigation equations, as well as the EKF and SR-UKF formulations to estimate the relative quantities of two spacecrafts using LOS observations, are derived by following similar approaches as in [6,11]. The general six degrees of freedom (DOF) relative equations of motion are expressed in the invariant tensor form, which are valid in any coordinate system [15].

The structure of this paper is as follows. In Section 2 we present several reference frames used to derive the equations of motion, then we give the derivation of the SR-UKF in detail in Section 3. The quaternion attitude parameter and relative attitude kinematics are introduced in Section 4. This is followed by relative position equations in Section 5 and the implementation details of the EKF and SR-UKF for relative navigation are provided in Section 6. In Section 7 the simulation results are presented to compare the performance of filters with each other. Finally, Section 8 contains some concluding remarks.

2. Reference frames and notations

Both inertial and non-inertial reference frames used to derive the spacecraft relative navigation equations are described in this section.

A reference frame can be created by defining the frame's origin and sense of direction. In this paper, all reference frames will have a right-handed sense in which the positive directions of each axis will form an orthogonal triplet. An inertial reference frame is one in which Newton's Laws of Motion apply. A commonly used inertial frame for near-Earth applications is the Earth-centered-inertial (ECI) whose origin is located at the center of the Earth. The frame's x-axis points towards the vernal equinox, the z-axis points towards the geographic north pole, and the y-axis completes the triplet. Inertial frame is identified by the superscript I. Two body-fixed reference frames are used to describe the vehicle body rotation with the origin located at the center of mass of the vehicle. The leader spacecraft body-fixed reference frame is identified by the superscript L, while the follower body-fixed reference frame is denoted by the superscript F.

The nomenclature used in this paper is consistent with [15]. Scalars are represented by lower-case characters, vectors by bold lower-case characters, and tensors by bold upper-case characters. A subscript is used to signify a point and a superscript is used to imply a frame. For instance, the displacement vector of point F with respect to (w.r.t.) point L is represented by $\boldsymbol{s}_{FL}$, the acceleration of point F w.r.t. frame I is represented by $\boldsymbol{a}_F^I$, and the angular velocity of body-fixed frame F w.r.t. leader frame L is given by ω^{FL}. $[*]^L$ represents the vector or tensor $*$ coordinated in L coordinate system. It should be noted that both the points and frames are denoted by slanted capital characters. The superscript T is reserved for transpose operator.

3. Square-root unscented Kalman filter

In this section, we give the derivation of the SR-UKF for nonlinear state estimation.

The general discrete-time nonlinear system model with purely additive process and observation noise is given by

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{1_{k-1}}) + \mathbf{v}_{k-1}$$
$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_{2_k}) + \mathbf{n}_k \tag{1}$$

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$ denotes the $n_x \times 1$ state vector and $\mathbf{y}_k \in \mathbb{R}^{n_y}$ the $n_y \times 1$ output observation vector. $\mathbf{v}_k \in \mathbb{R}^{n_x}$ is the $n_x \times 1$ state noise process vector and $\mathbf{n}_k \in \mathbb{R}^{n_y}$ the $n_y \times 1$ measurement noise vector. $\mathbf{u}_{1_k} \in \mathbb{R}^{n_{u_1}}$ is the $n_{u_1} \times 1$ state input vector and $\mathbf{u}_{2_k} \in \mathbb{R}^{n_{u_2}}$ the $n_{u_2} \times 1$ observation input vector. The mappings $\mathbf{f} : \mathbb{R}_{n_x} \times \mathbb{R}_{n_{u_1}} \mapsto \mathbb{R}_{n_x}$ and $\mathbf{h} : \mathbb{R}_{n_x} \times \mathbb{R}_{n_{u_2}} \mapsto \mathbb{R}_{n_y}$ represent the deterministic process and measurement models. It is assumed that $\mathbf{v}_k$ and $\mathbf{n}_k$ are uncorrelated zero-mean Gaussian noise processes with covariances given by $\mathbf{P}_{\mathbf{v}_k}$ and $\mathbf{P}_{\mathbf{n}_k}$.

The filter is initialized by the matrix square root of the state covariance via a Cholesky factorization. However, the propagated and updated Cholesky factor is then used in subsequent iterations to directly form the sigma points.

The entire algorithm is presented as follows:

(1) *Initialization*:

$$\hat{\mathbf{x}}_0 = E[\mathbf{x}_0], \quad \mathbf{S}_{\mathbf{x}_0} = \mathrm{chol}\{E[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^T]\}$$
$$\mathbf{S}_{\mathbf{v}_0} = \mathrm{chol}\{\mathbf{P}_{\mathbf{v}_0}\}, \quad \mathbf{S}_{\mathbf{n}_0} = \mathrm{chol}\{\mathbf{P}_{\mathbf{n}_0}\} \tag{2}$$