Acta Astronautica 66 (2010) 844–863

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Three-dimensional convection and unstable displacement of viscous fluids from strongly encumbered space

N.N. Smirnov *, V.F. Nikitin, V.R. Dushin, Yu.G. Phylippov, V.A. Nerchenko

Moscow M.V. Lomonosov State University, Moscow 119992, Russia

A R T I C L E I N F O

Article history:
Received 15 July 2009
Accepted 30 August 2009
Available online 26 September 2009

Keywords:
Convective flow
Displacement
Viscous fluid
Porous medium
Instability

A B S T R A C T

Purpose of the present paper is to investigate 3D instability effects in convective flows of viscous fluid displaced by a less viscous one from strongly encumbered space, and to determine characteristics of displacement quality. Fluids are assumed incompressible and miscible. Extensive direct numerical simulations are used to study the sensitivity of the displacement process to variation of values of the main governing parameters. Comparison with results of two-dimensional simulations enabled us to investigate the effect of aspect ratio on instability growth in viscous fluids displacement. A 1D model with two fitting parameters is created in order to simulate behavior of the cross section averaged parameters of the flow.

0. Introduction

The problem of fluid flows in essentially encumbered space are relevant to fluid flows in heat exchangers of different types, wherein flow channels are blocked up by different heat exchange elements. The present investigation is also relevant to studying the thermal fields established inside the spacecraft capsule under the flight conditions [1,2]. The great amount of the containers and complexity of the fluid–solid body interface is the main difficulty for the model elaboration. The traditional approach requires the calculation of fluid flow in a space of very complex form shaped containers and the boundary conditions should be formulated along the whole fluid–solid body interface. To avoid this difficulty an original approach has been worked out performing calculation. The fluid flow in the capsule was simulated using the model for fluid filtration in porous medium of changeable permeability. The mathematical model of fluid filtration through the medium with variable permeability is more suitable for the numerical calculations as it does not need the formulation of great amount of boundary conditions at the container surfaces and the calculation of flow in the space of a rather complex form between the containers. To avoid these difficulties, it is enough to determine the medium in the containers as a low permeable and the medium between them as high permeable [2]. Essentially different temperatures in fluid bring to viscosity variation, which in case of forced convection definitely present under microgravity conditions creates the situation of viscous fluid displacement by a less viscous one.

In frontal displacement of a more viscous fluid by a less viscous one Saffman–Taylor instability of the interface could result in formation of “fingers” of displacing fluid penetrating the bulk of the displaced one. The growth of fingers and their further coalescence could not be described by a linear analysis. Growth of fingers causes irregularity of the mixing zone thus affecting the displacement quality and heat exchange forecasts.

The problems of seepage flows were studied by many authors [3–12]. Investigating instability in miscible displacement differs greatly from that in immiscible fluids. The presence of a small parameter incorporating surface tension for immiscible fluids allows to determine theoretically the characteristic shape and width of viscous fingers [7,8], while

* Corresponding author. Tel.: +7 495 9391190; fax: +7 495 9394995.
E-mail addresses: ebifsun1@mech.math.msu.su, wonrims@inbox.ru (N.N. Smirnov).

0094-5765/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.actaastro.2009.08.028

in miscible fluids theoretical analysis allows to forecast the shape of the tips, but does not allow to determine the width of fingers, which remains a free parameter [5,6]. Numerical simulations of viscous fingering in miscible and immiscible displacement were carried out in [9,10]. Those papers contain an extensive bibliography on the history of the research as well. Numerical simulations [11] made it possible to explain new experimental results on the pear-shape of fingers and periodical separation of their tip elements from the main body of displacing fluid. Those separated blobs of less viscous fluid move much faster than the mean flow of the displaced viscous fluid [12].

The results of numerical simulations allowed to introduce dimensionless parameters characterizing the quality of displacement and the mixing flux induced by instability [12]. In the paper [13] the asymptotic behavior of miscible displacements in porous media was studied in the two limits, where a permeability-modified aspect ratio, became large or small, respectively.

The influence of inhomogeneity of porous matrix on displacement instability was investigated [13–15]. The modified Hele–Shaw cell containing regular and randomized obstacles was used to model and study the effect of inhomogeneity on displacement instability [12,14]. Results of numerical simulations as well as physical experiment showed that the presence of inhomogeneity of a definite length scale could stabilize unstable displacement and could destabilize a stable one [14].

Most of viscous fingering numerical simulations were performed for two-dimensional problems; one of the first classical publications was by Homsy [9,16]. This work used the spectral methods approach. In reality displacement and induced instability have a three dimensional nature. Papers [17–22] investigate 2D and 3D miscible displacement of fluids with account for gravity using the spectral methods, as in earlier works by Homsy.

First attempts to perform comparative analysis for the instabilities arising in displacement from 3D cells of different aspect ratios were performed in [23,36].

The present paper is aimed at numerical investigation of incompressible miscible fluids displacement in 3D geometry porous medium and studying the displacement scenarios being functions of aspect ratios and other dimensionless governing parameters.

For practical applications, such as simulating general heat exchange, often it is not important to have a detailed picture of viscous fingers development, rather then to have a quantitative estimate of the mixing flux induced due to displacement instability. Thus it is necessary to elaborate methods making it possible to simulate instability induced mixing within some integral approach, not sensitive to spatial resolution. A 1D model describing dynamic behavior of cross section averaged parameters is built with two fit parameters. Developing those model parameters for description of displacement quality and the mixing flux due to instability is also one of the goals of the present research. The regular porosity and permeability variation as shown in [11,12] could stabilize displacement (reduce the disturbances growth rate), which is important for stable operation of heat exchangers.

1. The problem statement

1.1. Physical description

The parallelepiped domain filled with porous media is regarded [23]. The following co-ordinate system is used: its origin is placed in the center of the inflow section. The axis Ox is directed along the domain towards the outflow section. The axes Oy and Oz are placed, respectively, vertically and sideward. The lateral surfaces bounding the domain are impermeable. Initially, almost all the domain is filled with the fluid to be displaced except for a tiny portion near the inflow cross-section. The displacing fluid flows via the cross-section at $x=0$; both fluids outflow via the section at $x=L$.

The pressure is assumed to be uniform both at the inflow and outflow cross-sections being not constant in time. We will study a constant rate problem, i.e. a fluid flux at the inflow section is set as a problem parameter. Both fluids are assumed incompressible; the porous skeleton is assumed immobile. The skeleton porosity is constant as well as the permeability. The gravity and other mass forces are neglected.

1.2. Scaling formulae

We choose the scaling factors denoted with a subscript s, as follows:

$$U_s = U, P_s = \mu_1 LU/K, t_s = L\phi/U, x_s = y_s = z_s = L,$$

$$V_s = W_s = U_s, \mu_s = \mu_1. \tag{1}$$

The following notations are used in (1): U is the mean velocity of filtration via inflow cross-section; L the length of the domain; μ_1 the displacing fluid viscosity; K the permeability; ϕ the porosity.

1.3. Governing equations

After the scaling is applied, the following dimensionless equations are obtained [12]

$$\frac{\partial s}{\partial t} + \nabla \cdot (s\mathbf{v}) = \nabla \cdot (\mathbf{D}(\mathbf{v}) \cdot \nabla s), \tag{2}$$

$$\mu(s)\mathbf{v} = -\nabla p, \tag{3}$$

$$\nabla \cdot \mathbf{v} = 0. \tag{4}$$

Eq. (2) states for the displacing fluid saturation dynamics, Eq. (3) is the generalized Darcy law, and Eq. (4) is a consequence of incompressibility of fluids and immobility of the porous skeleton. The following notations of dimensionless parameters are used in the system of governing Eqs. (2)–(4):

s	saturation of the displacing fluid, ranging from 0 to 1
$\mathbf{v}$	vector of the mean volumetric velocity of filtration
$\mathbf{D}$	dispersion tensor, depending on velocity
μ	effective fluids viscosity, depending on saturation
p	pressure