Acta Astronautica 66 (2010) 864–870

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Analytical approach using KS elements to high eccentricity orbit predictions including drag

Ram Krishan Sharma *, M. Xavier James Raj

Applied Mathematics Division, Vikram Sarabhai Space Centre, Thiruvananthapuram 695 022, India

ARTICLE INFO

Article history:
Received 21 December 2008
Received in revised form
2 April 2009
Accepted 2 September 2009
Available online 22 October 2009

Keywords:
Analytical non-singular theory
Orbit predictions
KS elements
Atmospheric drag
Oblate atmosphere
Density scale height variation

ABSTRACT

A new non-singular analytical theory for the contraction of high eccentricity satellite orbits under the influence of air drag is developed in terms of the KS elements using an oblate atmosphere with variation of density scale height with altitude. The series expansions include up to fourth power in terms of an independent variable Λ (function of the eccentric anomaly) and c (a small parameter dependent on the flattening of the atmosphere). Only two of the nine equations are solved analytically to compute the state vector and change in energy at the end of each revolution, due to symmetry in the equations of motion. It is observed that the analytically computed values of the semi-major axis (a) and eccentricity (e) match very well with the numerically integrated values up to 1000 revolutions over a wide range of the drag perturbed orbital parameters. Inclusion of the density scale height variation with altitude is found to increase the decay of the high eccentricity orbits up to eight percent. The theory can be used effectively for the orbital decay of aero-assisted orbital transfer orbits during mission planning.

1. Introduction

Aerocapture and aerogravity assist are two important space flight approaches that can enhance several planetary missions [1]. Aerobraking is the use of passes through a planet's atmosphere to slow down and lower altitude using the friction created from flying through the atmosphere. Generally the solar panels are used to provide the maximum drag in a symmetrical position so that control of the spacecraft can continue while it passes through the atmosphere. Repeated drag passes are used to shape the trajectory from its initial elliptical orbit into a desired circular orbit. Successive aerobraking depends upon precise navigation, accurate orbit prediction, knowledge of atmospheric conditions and an understanding of the limit of the forces that the spacecraft can withstand. Due to preliminary spacecraft limitations, aerobraking can involve a large number of drag passes over a long period. When there are concerns over spacecraft integrity (like with the Mars Global Surveyor solar array), aerobraking can take a longer time. Aerobraking, after an initial demonstration on Magellan mission at Venus, has now become the de facto approach for the Mars exploration programme with orbiters Mars Global Surveyor and Mars Odyssey having used this technique. NASA's Mars Reconnaissance Orbiter, a multipurpose spacecraft designed to conduct reconnaissance and exploration of Mars from orbit, attained Martian orbit in March 2006. In November 2006, after five months of aerobraking, it entered its final science orbit and began its science phase. Mars Global Surveyor, launched in November 1996, reached Mars in September 1997. It entered a highly elliptic orbit with periapsis and apoapsis of 260 km and 34,000 km, respectively. After performing a series of orbit changes to lower the periapsis of its orbit at an altitude of 110 km, it used

* Corresponding author. Tel.: +91 471 2565629; fax: +91 471 2564031.
E-mail addresses: ramkrishansharma@gmail.com, rk_sharma@vssc.gov.in (R.K. Sharma).

0094-5765/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.actaastro.2009.09.010

aerobraking technique for a period of four months to lower the apoapsis to nearly 450 km.

These highly elliptic orbits have large variations in velocity and in perturbations during a revolution and the computational efficiency of the numerical methods also decreases. During the planning of such missions, the analytical solutions are preferred as they represent a manifold of solutions for a large domain of initial conditions and find indispensable application to mission planning and qualitative analysis. The KS total-energy element equations are found to be a powerful method for numerical solution with respect to different type of perturbing forces (Stiefele and Scheifele [2]; Sharma and Mani [3]). Sharma [4–6] had utilized these equations to generate non-singular analytical solutions for orbit predictions for low-Earth orbits with air drag perturbation using oblate exponential and diurnally oblate exponential atmospheric models. In Sharma [7] by using a new independent variable λ in place of the eccentric anomaly E, introduced by Sterne [8] as $\cos E=\{1-\lambda^2/z\}$, $z=ae/H$, H being the density scale height, an analytical theory for orbit predictions was generated in terms of the KS elements using spherically symmetrical exponential atmosphere. The KS element equations were also utilized to generate an analytical solution for orbit predictions with oblate exponential atmosphere in [9]. In all the above studies, the variation of H with altitude was kept constant.

By including the variation of density scale height with altitude in [9], an analytical solution was described in [10]. It may be pointed out that in both the above papers only the general methodology to compute the analytical solution were presented and did not contain the explicit analytical theories. All the expressions required for the analytical solution in [10], are evaluated using MAXIMA software [11], which is a free computer algebra system based on a 1982 version of the Macsyma, written in Common Lisp and released under the General Public Licence (GNN). This paper contains the complete analytical solution. Terms up to fourth-order in $\Lambda=\lambda^2$ and c (a small parameter dependent on the flattening of the atmosphere) are retained. The semi-major axis and eccentricity, obtained from the theory, are compared with the numerically integrated values up to 1000 revolutions. It is observed that the percentage error increases marginally with the increase in the revolution number. It is also noted that the inclusion of the density scale height variation with altitude increases the decay of the orbits significantly, which is found to be up to 8 percent.

2. Equations of motion

Following the terminology, notations and procedure as in [2,4,7,8], the equations of motion of a satellite in terms of the KS elements under the effect of the atmospheric drag force

$$\vec{P}=-\tfrac{1}{2}\rho\delta|\vec{v}|\vec{v}, \tag{1}$$

per unit mass acting on a satellite of mass m with velocity $\vec{v}$ are

$$\frac{dw}{dE}=-\frac{\rho\delta|\vec{v}|}{16r}[f_0+f_1\cos 2E+f_2\sin 2E], \tag{2}$$

$$\begin{aligned}\frac{d\varsigma_i}{dE}=\frac{\rho\delta|\vec{v}|}{16\omega}&[(g_{0i}+g_{1i}\cos E+g_{2i}\sin E+g_{3i}\cos 2E\\&+g_{4i}\sin 2E)+\frac{1}{r}(h_{0i}+h_{1i}\cos E+h_{2i}\sin E\\&+h_{3i}\cos 2E+h_{4i}\sin 2E+h_{5i}\cos 3E+h_{6i}\sin 3E)],\\&i=1-8,\end{aligned} \tag{3}$$

with

$$\zeta_i=\alpha_i \quad\text{for } i=1-4,\quad\text{and}\quad \zeta_i=\beta_{i-4}\quad\text{for } i=5-8,$$

where

$$\begin{aligned}f_0&=2[d_{11}^2+d_{12}^2+d_{21}^2+d_{22}^2+d_{31}^2+d_{32}^2],\\ f_1&=2[d_{11}^2-d_{12}^2+d_{21}^2-d_{22}^2+d_{31}^2-d_{32}^2],\\ f_2&=4[d_{11}d_{12}+d_{21}d_{22}+d_{31}d_{32}],\end{aligned}$$

$$\begin{aligned}d_{11}&=-\alpha_1\beta_1+\alpha_2\beta_2+\alpha_3\beta_3-\alpha_4\beta_4,\\ d_{12}&=\tfrac{1}{2}[\alpha_1^2-\alpha_2^2-\alpha_3^2+\alpha_4^2-(\beta_1^2-\beta_2^2-\beta_3^2+\beta_4^2)],\\ d_{21}&=-\alpha_1\beta_2-\alpha_2\beta_1+\alpha_3\beta_4-\alpha_4\beta_3,\\ d_{22}&=\alpha_1\alpha_2-\alpha_3\alpha_4-\beta_1\beta_2+\beta_3\beta_4,\end{aligned}$$

$$\begin{aligned}d_{31}&=-(\alpha_1\beta_3+\alpha_2\beta_4+\alpha_3\beta_1+\alpha_4\beta_2),\\ d_{32}&=\alpha_1\alpha_3+\alpha_2\alpha_4-\beta_1\beta_3-\beta_2\beta_4,\end{aligned}$$

$$\begin{aligned}g_{0i}&=d_{11}q_{11}^{(i)}+d_{12}q_{21}^{(i)}+d_{21}q_{12}^{(i)}+d_{22}q_{22}^{(i)}+d_{31}q_{31}^{(i)}+d_{32}q_{23}^{(i)},\\ g_{1i}&=2\{d_{11}q_{01}^{(i)}+d_{21}q_{02}^{(i)}+d_{31}q_{03}^{(i)}\},\\ g_{2i}&=2\{d_{12}q_{01}^{(i)}+d_{22}q_{02}^{(i)}+d_{32}q_{03}^{(i)}\},\\ g_{3i}&=d_{11}q_{11}^{(i)}-d_{12}q_{21}^{(i)}+d_{21}q_{12}^{(i)}-d_{22}q_{22}^{(i)}+d_{31}q_{13}^{(i)}-d_{32}q_{23}^{(i)},\\ g_{4i}&=d_{11}q_{21}^{(i)}+d_{12}q_{11}^{(i)}+d_{21}q_{22}^{(i)}+d_{22}q_{12}^{(i)}+d_{31}q_{23}^{(i)}+d_{32}q_{13}^{(i)},\end{aligned}$$

$$\begin{aligned}&h_{0i}=f_0q_{04}^{(i)},\quad h_{1i}=\tfrac{1}{2}[(2f_0+f_1)q_{14}^{(i)}+f_2q_{24}^{(i)}],\\ &h_{2i}=\tfrac{1}{2}[(2f_0-f_1)q_{24}^{(i)}+f_2q_{44}^{(i)}],\quad h_{3i}=f_1q_{04}^{(i)},\\ &h_{4i}=f_2q_{04}^{(i)},\quad h_{5i}=\tfrac{1}{2}[f_1q_{14}^{(i)}-f_2q_{24}^{(i)}],\quad h_{6i}=\tfrac{1}{2}[f_1q_{24}^{(i)}-f_2q_{14}^{(i)}],\end{aligned}$$

with

$$\delta=FA_{eff}\,C_D/m,\quad F=[1-r_{p0}\overline{\Lambda}\cos i_0/v_{p0}]^2,$$

where ρ is the atmospheric density, ω is the angular frequency, $\overline{\Lambda}$ is the rotational rate of the atmosphere about the Earth's axis, r_{p0} is the initial perigee radius, i_0 is initial inclination, v_{p0} is the velocity at the initial perigee, C_D and A_{eff} are, respectively, the drag coefficient and the effective area of the satellite. The initial KS element parameters $q_{jk}^{(i)}$ for j=0–2, k=1–4 are provided in [4].

3. Oblate exponential atmosphere

In an atmosphere of constant scale height H and ellipticity ε (taken equal to the Earth's ellipticity, 0.00335 [12]), the density ρ may be written as

$$\rho=\rho_P\exp\left(-\frac{r-\sigma}{H}\right), \tag{4}$$

where

$$\sigma=r_P[1-\varepsilon\sin^2 i\{\sin^2(\omega+\theta)-\sin^2\omega\}+O(\varepsilon^2)], \tag{5}$$