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Acta Astronautica 64 (2009) 810–831

ACTA ASTRONAUTICA

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Asymptotic theory for thrusting, spinning-up spacecraft maneuvers

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Received 3 November 2006; accepted 16 December 2008
Available online 5 February 2009

Abstract

We consider the problem of a spacecraft subjected to constant body-fixed forces and moments about all three axes during a spinning-up, thrusting maneuver. In applications, undesired forces and moments can arise due to thruster imbalances and misalignments and to center-of-mass offset. In previous works, approximate analytical solutions have been found for the attitude motion, and for the change in inertial velocity and inertial position. In this paper we find asymptotic and limiting-case expressions which we derive from the analytic solutions, in order to obtain simplified, practical formulas that lend insight into the motion. Specifically, we investigate how the motion evolves (1) as time grows without bound and (2) for geometric cases of the sphere, the thin rod, and the thin plate. Closed-forms or upper-bound limits are provided for angular velocities, Eulerian angles, angular momentum pointing error, transverse and axial velocities, and transverse and axial displacements. Summaries for the asymptotic limits (for zero initial conditions) are provided in tabular form. Results are verified by numerical simulations.

Keywords: Asymptotic theory; Thrusting; Spinning-up spacecraft; Rigid body motion

1. Introduction

The problem of spinning rockets and spacecraft has been studied for more than half a century et al. [1–52]. During the period from 1947 to 1961, Rosser [1], Jarmolow [3], Davis et al. [4], Buglia et al. [5], and Martz [6] analyzed the motion of a spinning rocket when the mass properties change with time. In 1965, Armstrong [7] extended the analytical work by developing an asymptotic theory for the thrusting, spinning, and the spinning-up spacecraft problem. Because of the lack of complete analytic solutions his study was limited to some special cases. Following Armstrong's work, there has been significant effort to describe the attitude and translational motion of a spinning spacecraft. The analytical developments by Likins [9], Cochran et al. [10,22,23], Junkins et al. [11,18], Larson and Likins [12,14], Kraige et al. [16,17], Price [21], Longuski et al. [19,25,32,34–36,39], Van der Ha [27], Winfree and Cochran [28], Kane and Levinson [29], Tsiotras and Longuski [33,40], Gick et al. [44–46], Randall et al. [38], Livneh and Wie [42], Sookgaew and Eke [48], focused on rotational motion and stability analyses of a spinning spacecraft. In recent years, the analytical theory of translational motion of a spinning rigid body has been more developed by Longuski et al. [49], Beck and Longuski [37], and Ayoubi and Longuski [50–52].

Analytical solutions give insight into the behavior of the motion by providing explicit expressions for periodic motion, secular effects and asymptotic limits. Identifying periodic and secular terms or determining the asymptotic limits arising from a complete but

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doi:10.1016/j.actaastro.2008.12.015

complicated analytical theory lead to simple closed-form practical solutions. Also, considering the geometrically limiting cases of a sphere, a thin rod, and a flat disk can drastically reduce the number of terms in the full-blown theory. The distillation of such simplified expressions can give the practical engineer a handle in understanding the essential motion—a back-of-the envelope calculation, a sanity check. In this paper we present an asymptotic theory for the thrusting, spinning-up spacecraft problem based on the results of Tsiotras and Longuski [40] and Ayoubi and Longuski [51,52].

2. The thrusting, spinning-up problem

2.1. Background

Let us consider the motion of a spinning rigid body in an inertial reference frame, XYZ, as shown in Fig. 1.

We assume that the body-fixed reference frame, xyz, is located at the center of mass and aligned with the principal axes of the rigid body. Euler's equations of motion [53]—which governs the rotational motion of a rigid body—can be written in the following form:

$$\dot{\omega}_x(t) = M_x/I_x - [(I_z - I_y)/I_x]\omega_y\omega_z \tag{1}$$

$$\dot{\omega}_y(t) = M_y/I_y - [(I_x - I_z)/I_y]\omega_z\omega_x \tag{2}$$

$$\dot{\omega}_z(t) = M_z/I_z - [(I_y - I_x)/I_z]\omega_x\omega_y \tag{3}$$

where ω_x, ω_y, and ω_z are components of the angular velocity vector, I_x, I_y, and I_z are constant principal moments of inertia, and M_x, M_y, and M_z are moments along the body-fixed x, y, and z axes, respectively. We

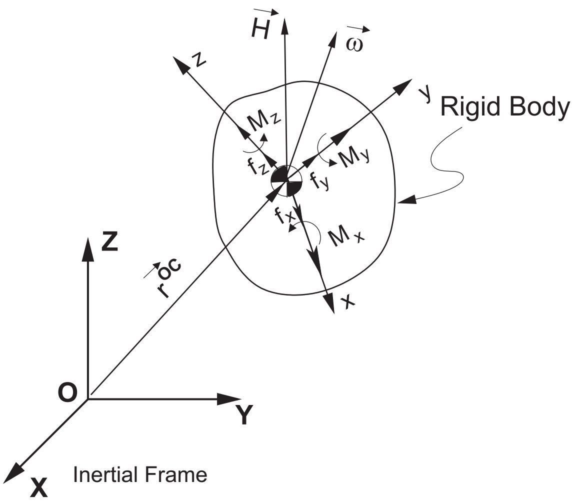

Fig. 1. A spinning rigid body with body-fixed forces and moments in an inertial frame.

assume the moments are constant. For an axisymmetric, nearly axisymmetric, and under some circumstances for an asymmetric rigid body (when the second term is negligible in compare to the first term on the right-hand side of Eq. (6)), Eqs. (1)–(3) reduce to

$$\dot{\omega}_x(t) = M_x/I_x - [(I_z - I_y)/I_x]\omega_y\omega_z \tag{4}$$

$$\dot{\omega}_y(t) = M_y/I_y - [(I_x - I_z)/I_y]\omega_z\omega_x \tag{5}$$

$$\dot{\omega}_z(t) \approx M_z/I_z \tag{6}$$

We assume that the spin axis is the z axis which may be either the minimum or maximum moment-of-inertia axis. (We do not consider spinning about the intermediate axis, which is unstable.)

Eq. (6) can be integrated as

$$\omega_z(t) = (M_z/I_z)t + \omega_{z0}, \quad \omega_{z0} \triangleq \omega_z(0) \tag{7}$$

The kinematic equations describe the orientation of a rigid body with respect to a reference frame. For a Type-I Euler sequence 3–1–2 (ϕ_z, ϕ_x, ϕ_y) are given by the following equations [54]:

$$\dot{\phi}_x = \omega_x \cos\phi_y + \omega_z \sin\phi_y \tag{8}$$

$$\dot{\phi}_y = \omega_y - (\omega_z \cos\phi_y - \omega_x \sin\phi_y)\tan\phi_x \tag{9}$$

$$\dot{\phi}_z = (\omega_z \cos\phi_y - \omega_x \sin\phi_y)\sec\phi_x \tag{10}$$

For spin-stabilized spacecraft and rockets, we can assume that ϕ_x, ϕ_y, and $\phi_y\omega_x$ are small. Thus, Eqs. (8)–(10) can be simplified as

$$\dot{\phi}_x = \omega_x + \omega_z\phi_y \tag{11}$$

$$\dot{\phi}_y = \omega_y - \phi_x\omega_x \tag{12}$$

$$\dot{\phi}_z = \omega_z \tag{13}$$

The Eulerian angles ϕ_z, ϕ_x, and ϕ_y are illustrated in Fig. 2.

Integrating Eq. (13) yields

$$\phi_z = \frac{1}{2}\frac{M_z}{I_z}t^2 + \omega_{z0}t + \phi_{z0}, \quad \phi_{z0} \triangleq \phi_z(0) \tag{14}$$

With the presence of constant body-fixed forces (f_x, f_y, f_z) during thrusting and spinning-up maneuver (due primarily to thruster imbalance and misalignment, and center-of-mass offset), the translational motion of the rigid body can be described by

$$\begin{Bmatrix} \dot{v}_X(t) \\ \dot{v}_Y(t) \\ \dot{v}_Z(t) \end{Bmatrix} = [A]_{312} \begin{Bmatrix} f_{x/m} \\ f_{y/m} \\ f_{z/m} \end{Bmatrix} \tag{15}$$