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Acta Astronautica 64 (2009) 288 – 294

ACTA ASTRONAUTICA

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Fluid flow interaction with an obstacle near free surface

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Received 6 August 2008; accepted 2 September 2008
Available online 23 October 2008

Abstract

The two-dimensional problem of thin obstacle interaction with compressible fluid flow near free surface under microgravity was regarded. The fluid was assumed occupying infinite semi-space, gravity was neglected as compared with fluid inertia. The obtained exact solution shows that for relatively thin layer the resistance force does not depend on the layer thickness, but depend on body length and inclination angle, and surpasses twice the respective values for the infinite medium, which could be explained by additional wave resistance arising in the presence of free surface.

Keywords: Compressibility; Fluid; Flow; Free surface; Motion; Resistance; Lift force; Singularity

1. Introduction

The problem of fluid streaming rigid bodies in the presence of free surface of fluid of infinite and finite depth was regarded within the frames of linear [1–4] and non-linear [5–7] statements. The problem could be regarded in two-dimensional case as fluid flow interacting with a plane located at a definite depth under free surface (Fig. 1). Under the conditions this motion takes place for a relatively long time the assumption of steady-state flow field is valid.

2. Problem statement

The two-dimensional problem of fluid streaming thin body motion of infinite span with velocity parallel to the free surface is regarded under the assumption of flow separation from the upper surface of the body (Fig. 1). Pressures on free surface and in the cavity are assumed equal to vapor ambient pressure. Fluid is assumed to be ideal, depth-infinite, mass forces-negligibly small, flow field-plane. Velocity field in fluid is assumed to be potential

$$\vec{V} = \vec{V}_0 + \operatorname{grad}\varphi, \tag{1}$$

fluid will be regarded as linear compressible

$$P = P_0 + a^2(\rho - \rho_0), \quad a^2 = \left(\frac{\mathrm{d}P}{\mathrm{d}\rho}\right)_{\rho_0}, \tag{2}$$

$$P = P(\rho), \quad \mathrm{d}s = 0 \to \mathrm{d}P = \left(\frac{\mathrm{d}P}{\mathrm{d}\rho}\right)_{\rho_0} \mathrm{d}\rho,$$

where $\varphi(x, y, t)$—disturbance velocity potential, P, ρ—fluid pressure and density, P_0, ρ_0—pressure and density in quiescent fluid, a—sonic velocity.

Fluid flow satisfies continuity equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \operatorname{div}\vec{V} = 0, \tag{3}$$

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doi:10.1016/j.actaastro.2008.09.003

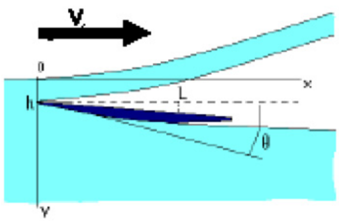

Fig. 1. Thin body streaming by a compressible fluid at a depth h, constant velocity V_0 parallel to free surface and inclination angle θ.

pressure is determined by Cauchy–Lagrange integral [10]

$$\frac{\partial\varphi}{\partial t}+\frac{(\mathrm{grad}\,\varphi)^2}{2}+\int\frac{\mathrm{d}P}{\rho}=c(t). \tag{4}$$

Flow induced variations of density and velocity are considered small values.

$$\rho'/\rho=(\rho-\rho_0)/\rho\ll 1;\quad u_x/V_0\ll 1;\quad u_y/V_0\ll 1,$$

where u_x, u_y—disturbance velocity components.

$$\int\frac{\mathrm{d}P}{\rho}=\int\frac{\mathrm{d}P}{\rho'+\rho_0}\approx\int\frac{\mathrm{d}P}{\rho_0}=\frac{P-P_0}{\rho_0}\to P=P_0-\rho_0\frac{\partial\varphi}{\partial t}-\frac{1}{2}\rho_0V_0^2,$$

$$\mathrm{d}P=a^2\,\mathrm{d}\rho$$
$$\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{\partial P}{\partial t}+(V_0+u_x)\frac{\partial P}{\partial x}+u_y\frac{\partial P}{\partial y}=\frac{\partial P}{\partial t}+V_0\frac{\partial P}{\partial x}.$$

Then it follows from continuity equation (3), integral (4) and relationships (1), (2) neglecting small values of the orders higher than one, flow potential φ under the condition of steady-state flow satisfies the equation

$$V_0^2\frac{\partial^2\varphi}{\partial x^2}=a^2\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right), \tag{5}$$

and fluid pressure is determined

$$P-P_0=\rho_0V_0\frac{\partial\varphi}{\partial x}. \tag{6}$$

Boundary conditions should be satisfied on free surfaces and on the body surface contacting fluid. On free surfaces constant pressure is assumed, on the fluid-body contact streaming condition of the equality of normal velocity component. $\vec{n}=(-\sin\theta;\cos\theta)$,

$$V_n=\bar{V}\vec{n}=-(V_0+u_x)\sin\theta+u_y\cos\theta=-(V_0+u_x)\theta+u_y=0,$$

which yields $u_y=(V_0+u_x)\sin\theta$.

The obstacle being thin and inclination angle being small all disturbances could be considered small, and boundary conditions take the form

$$\begin{aligned}&y=0,\quad P-P_0=0;\\&y=h^-,\quad 0<x\ \ P-P_0=0;\\&y=h^+,\quad 0<x<L\ \ u_y=\frac{\partial\varphi}{\partial y}=V_0\cdot\sin\theta;\\&y=h^+,\quad L<x\ \ P-P_0=0.\end{aligned} \tag{7}$$

Substituting in Eq. (7) dynamical equation (6) boundary conditions look as follows:

$$\begin{aligned}&y=0,\quad\frac{\partial\varphi}{\partial x}=0;\quad y=h^-,\quad 0<x\ \ \frac{\partial\varphi}{\partial x}=0;\\&y=h^+,\quad 0<x<L\ \ \frac{\partial\varphi}{\partial y}=V_0\cdot\sin\theta;\\&y=h^+,\quad L<x\ \ \frac{\partial\varphi}{\partial x}=0.\end{aligned} \tag{8}$$

Thus Eq. (5) with boundary conditions (8) present a closed form statement of the problem.

3. Problem solution

We assume the flow to be subsonic. Then on introducing dimensionless parameter $\alpha=\sqrt{1-M^2}$, where $M=V_0/a$—Mach number and dimensionless functions and variables

$$\varphi^*=\frac{\pi\varphi}{ah};\quad p^*=\frac{P-P_0}{\rho_0a^2};\quad l=\frac{L\pi}{h\alpha};\quad x^*=\frac{\pi x}{\alpha h};\quad y^*=\frac{\pi y}{h}, \tag{9}$$

equations and boundary conditions take the form

$$\begin{aligned}&\frac{\partial^2\varphi^*}{\partial x^{*2}}+\frac{\partial^2\varphi^*}{\partial y^{*2}}=0,\quad p^*=\frac{M}{\alpha}\frac{\partial\varphi^*}{\partial x^*},\\&y^*=0,\quad\frac{\partial\varphi^*}{\partial x^*}=0;\quad y^*=\pi^-,\quad 0<x^*\ \ \frac{\partial\varphi^*}{\partial x^*}=0;\\&y^*=\pi^+,\quad 0<x^*<l\ \ \frac{\partial\varphi^*}{\partial y^*}=M\cdot\gamma(x^*)\frac{1}{\alpha};\\&y^*=\pi^+,\quad l<x^*\ \ \frac{\partial\varphi^*}{\partial x^*}=0.\end{aligned} \tag{10}$$

$$\sin\theta\approx tg\,\theta=\frac{\mathrm{d}y}{\mathrm{d}x}.\quad tg\,\theta=\frac{1}{\alpha}\frac{\mathrm{d}y^*}{\mathrm{d}x^*}=\frac{1}{\alpha}\gamma(x^*).$$

In successive derivations star in dimensionless value symbols will be omitted. The problem is reduced to developing analytical function in the domain $y>0$ with a