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On the magnetic attitude control for spacecraft via the ε-strategies method

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Abstract

We develop a new approach to stabilization problems based on a combination of the Lyapunov functions method with local controllability properties. The stabilizability is understood in the sense of ε-strategies introduced by Pontryagin in the frame of differential games theory. To illustrate the possibilities of our approach we consider a satellite with two magnetic coils directed along its principal inertia axes. Its circular orbit is neither polar nor equatorial. We show that there exists an ε-strategy stabilizing an Earth pointing satellite, whenever the deviations from the equilibrium position are small enough.

Keywords: Attitude control; Satellite control; Time-varying system; Periodic motion; Stability

1. Introduction

Attitude control system (ACS) is of great importance for a spacecraft mission success. The attitude control of small satellites is usually fulfilled by passive or semi-passive ACSs. These systems use the satellite interaction with the gravitational and magnetic field of the planet, atmospheric drag and the gyroscopic properties of spinning bodies. The general approaches to obtain main parameters of passive ACSs are presented in [1], for example. These satellite orientation methods are sufficient if the satellite does not require complex reorientation maneuvers during the flight. Otherwise active ACSs have to be used. From the simplicity point of view the most reliable and light ACSs are active magnetic systems [2,3]. The main difficulty in the implementation of such a system and in the proof of its normal operation consists in the fact that the control torque lies in the plane orthogonal to the magnetic field vector. The attitude stabilization systems with magnetic actuators for Earth pointing spacecraft on a circular orbit have been the subject of extensive studies during the last years (see the survey [3] and the references therein). Among numerous techniques developed to solve this problem the most natural one is the use of Lyapunov functions method combined with the Krasovski–LaSalle theorem [2]. The Lyapunov function is used to construct a stabilizer depending on current time and

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position of the system. Then, the Krasovski–LaSalle theorem is applied to prove the asymptotic stability of the equilibrium position. However, a rigorous verification of the Krasovski–LaSalle theorem conditions faces serious technical difficulties which can hardly be overcome even with the help of systems for symbolic computations, like Mathematica or Maple. In this paper we suggest an alternative way to solve stabilization problems. This method is based on a combination of the Lyapunov functions method with local controllability conditions. In many situations this method admits a rigorous mathematical justification and leads to effective numerical methods. The stabilizability is understood in a general sense. We use ε-strategies introduced by Pontryagin in the frame of differential games theory [4]. According to this approach the stabilizing control is constructed as a function of time defined in a small time interval and not as a feedback. From the practical point of view, ε-strategy is similar to stabilizer which depends on the time and position only, because it usually is implemented as a generator of piecewise constant controls. However, the use of this approach helps to overcome serious mathematical difficulties and is more effective in applications. The numerical implementation of this method is based on the construction of multistep reachability sets [5]. Note that the ideas from the differential games theory where also used in [6] to solve another problem from stabilization theory. To illustrate the possibilities of our approach we consider a satellite with two magnetic coils directed along its principal inertia axes. Its circular orbit is neither polar nor equatorial. We show that there exists an ε-strategy stabilizing an Earth pointing satellite, whenever the deviations from the equilibrium position are small enough. One coil is not sufficient to guarantee such a stabilization [7]. Throughout this paper we denote by R the set of real numbers and by R^n the usual n-dimensional space. The inner product of two vectors $x, y \in R^n$ is expressed by $\langle x, y\rangle$. The interior of a set $C \subset R^n$ is denoted by $\operatorname{int} C$. If A is a real matrix, then the transposed matrix is denoted by A^*. By $x \times y$ we shall denote the vector product of the vectors $x, y \in R^3$.

2. Stabilization via ε-strategies

Consider a control system

$$\dot{x} = f(t, x, u), \quad u \in U. \tag{1}$$

Assume that $0 \in \operatorname{int} U \subset R^k$ and a function $f : R \times R^n \times U \to R^n$, satisfies the following conditions: it is sufficiently smooth, $f(t, 0, 0) = 0$, $t \in R$, for all $(t, x) \in R \times R^n$, the set $f(t, x, U)$ is compact and convex, there exists $M > 0$ such that $\|f(t, x, u)\| \leqslant M$, for all $(t, x, u) \in R \times R^n \times U$, there exists $T > 0$ such that $f(t + T, x, u) = f(t, x, u)$, for all $(t, x, u) \in R \times R^n \times U$. Let $u(t) \in U$, $t \geqslant t_0$, be a measurable bounded control. The solution to the Cauchy problem $\dot{x}(t) = f(t, x(t), u(t))$, $t > t_0$, $x(t_0) = x_0$, is denoted by $x(\cdot, t_0, x_0, u(\cdot))$, and the symbol $X(t_0, x_0)$ is used for the set of all solutions. Consider the solution $\hat{x}(\cdot) = \hat{x}(\cdot, t_0, x_0, 0) \in X(t_0, x_0)$ corresponding to the control $u(t) \equiv 0$. Set $A(t) = \nabla_x f(t, \hat{x}(t), 0)$ and $B(t) = \nabla_u f(t, \hat{x}(t), 0)$. Introduce a linearization of control system (1) along the trajectory $\hat{x}(\cdot)$: $\dot{\bar{x}}(t) = A(t)\bar{x}(t) + B(t)w(t)$, $w(t) \in R^k$. The set of its solutions $\bar{x}(\cdot, t_0, \bar{x}_0, w(\cdot))$ is denoted by $\bar{X}(t_0, x_0)$. Define the reachability sets $\mathbf{R}(t_1, t_0, x_0) = \{x(t_1, t_0, x_0, u(\cdot)) \in R^n | x(\cdot, t_0, x_0, u(\cdot)) \in X(t_0, x_0)\}$ and $\bar{\mathbf{R}}(t_1, t_0, \bar{x}_0) = \{\bar{x}(t_1, t_0, \bar{x}_0, u(\cdot)) \in R^n | \bar{x}(\cdot, t_0, \bar{x}_0, u(\cdot)) \in \bar{X}(t_0, \bar{x}_0)\}$. Recall a well-known result from the control theory (see [8], for example).

Theorem 1. *The linearized system is controllable on the interval* $[t_0, t_1]$, *i.e.* $\bar{\mathbf{R}}(t_1, t_0, 0) = R^n$, *if and only if, only the trivial solution* $\bar{x}^*(t) \equiv 0$ *to the adjoint differential equation* $\dot{\bar{x}}^*(t) = -A^*(t)\bar{x}^*(t)$, $t \in [t_0, t_1]$, *satisfies the orthogonality condition* $B^*(t)\bar{x}^*(t) = 0$, $t \in [t_0, t_1]$. *The condition* $\bar{\mathbf{R}}(t_1, t_0, 0) = R^n$ *implies that control system* (1) *is controllable around the trajectory* $\hat{x}(\cdot)$ *on the interval* $[t_0, t_1]$, *i.e.* $\hat{x}(t_1) \in \operatorname{int} \mathbf{R}(t_1, t_0, x_0)$.

Let $V : R^n \to R$ be a sufficiently smooth function satisfying the conditions $V(0) = 0$ and $V(x) > 0$, $x \neq 0$. Its Lipschitz constant is denoted by L_V.

By ε-strategy σ we mean a map $(t_0, x_0) \to (\varepsilon, u(\cdot)) \in R^+ \times L_\infty([t_0, t_0 + \varepsilon], U)$. An ε-strategy σ generates a trajectory defined in the intervals $[t_0, t_0 + \varepsilon_1]$, $[t_0 + \varepsilon_1, t_0 + \varepsilon_1 + \varepsilon_2]$, etc. This trajectory is denoted by $x(t, t_0, x_0, \sigma)$.

Theorem 2. *Assume that there exist a neighborhood* Ξ *of the point* $x = 0$ *and* $\varepsilon_0 > 0$ *such that for all* $x_0 \in \Xi$, $t_0 \in R$ *and* $\varepsilon \in]0, \varepsilon_0[$ *there is a measurable bounded control* $u(t) \in U$, $t \in [t_0, t_0 + \varepsilon]$, *satisfying* $V(x(t_0 + \varepsilon, t_0, x_0, u(\cdot))) < V(x_0)$. *Then there exist a neighborhood* Ξ_1 *of the point* $x = 0$ *and an ε-strategy* σ *defined in* $R \times \Xi_1$, *such that for any* $t_0 \in R$, *the equality* $\lim_{t \to +\infty} x(t, t_0, x_0, \sigma) = 0$ *holds.*

Proof. Consider a monotone decreasing sequence V_m which tends to zero when m goes to infinity. Let $\Xi_m = \{x : V(x) < V_m\}$ and $\alpha_m = (V_{m-1} - V_m)/(2L_V M)$, $m = 1, 2, \ldots$. Without loss of generality, $\alpha_m < \varepsilon_0$. The ε-strategy σ is defined as follows. Let $(t_0, x_0) \in R \times \Xi_{m-1} \backslash \Xi_m$. The pair $(\varepsilon, u(\cdot))$ is a solution to