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Conduction heat transfer in semi-infinite and infinite regions with discrete heat sources

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Abstract

The steady state temperature distribution in semi-infinite slabs and infinite quadrants due to two- and three-dimensional discrete heating sources shaped in the form of thin electric current carrying wires has been determined. The temperature field is obtained using Green's function integral techniques. The solutions obtained here are compared with numerical solutions obtained from a commercial software package. It is shown that for the cases considered here we get closed form solutions or solutions in the form of simple numerically calculable integrals which are far superior to numerical methods in terms of elegance and labor involved for parametric studies. The behavior of the non-dimensional temperature with the various relevant parameters is discussed.

Keywords: Conduction heat transfer; Discrete heat sources; Integral techniques

1. Introduction

Discrete electric current heaters are often used in spacecrafts to maintain the various components operating within safe operating temperatures. The thermal control subsystem of the spacecraft activates these heaters when the spacecraft is in the shadow of the earth [1]. The problem of heat conduction in semi-infinite slabs and quadrants is also of fundamental importance in applied science and engineering. In this work we consider the problem of determining the temperature distribution in semi-infinite slabs and infinite quadrants with discrete heat generation sources in the form of thin electric current carrying wires. The classical treatment of heat conduction in solids by Carslaw and Jaeger [2] has been followed in a more modern format by Arpaci [3]. The specific problem of temperature distribution and thermal resistance determination in spacecraft mounting plates with regional heating sources has been studied by Venkataraman et al. [4], Venkataraman and Egalon [5] and Venkataraman and

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Nomenclature

$\vec{x}$	position vector of field point
$\vec{x}'$	position vector of source point
$(\bar{x}, \bar{y}, \bar{z})$	non-dimensional location of field point
k	thermal conductivity of material
$T(\vec{x})$	temperature at position $\vec{x}$
$Q(\vec{x})$	internal heat generation rate per unit volume (W/m^3)
$G(\vec{x}, \vec{x}')$	Green's function corresponding to $(\vec{x}, \vec{x}')$
$\delta(\vec{x} - \vec{x}')$	Dirac delta function
dS	infinitesimal surface area
$\hat{n}'$	unit outward normal to dS
$H(x)$	Heaviside step function
$\bar{T}$	non-dimensional temperature
a	distance a of the heating source to the vertical axis
$\bar{a}$	non-dimensional source location
(r, θ)	radial and angular coordinates of field point for circular ring heating source
(r', θ')	radial and angular coordinates of source point for circular ring heating source
$\bar{r}$	non-dimensional radius
b	distance a of the heating source to the vertical axis
$\bar{b}$	non-dimensional source location
λ	strength of the heat generation source per unit length and per unit depth
$\bar{\lambda}$	non-dimensional heat generation source strength
Λ	heat generation rate per unit length of the source (W/m)
$\bar{\Lambda}$	non-dimensional source strength

Sepulveda [6], where additional previous literature has been cited. Beck et al. [7] have summarized the use of Green's function methods for heat conduction problems basically using eigen function expansion techniques. Shendeleva et al. [8] and Shendeleva [9] discussed the temperature fields generated by instantaneous line heat sources and interfacial heat sources in a medium consisting of two half spaces of different thermal properties using Green's functions and Cagniard–de Hoop technique. While these methods use Green's functions obtained from the differential equations or from eigen function expansions, Venkataraman et al. [10] used Green's functions obtained by a more physical approach using the method of images. They used an integral approach and obtained solutions for circular plates with discrete heaters in the form of a circular arc heater and a spiral heater as well as a sphere with a line heating source running along a diameter. They showed that for these cases the method yields the results in a closed form or in the form of simple numerically calculable integrals which are more elegant and ideally suited for making parametric studies in comparison with numerical methods such as finite differences or finite elements. The objective of the present paper is to extend the work of Ref. [10] to semi-infinite slabs and infinite quadrants with various discrete heat generation sources. Although we have considered many different forms of sources, we present here only four cases as follows:

(a) square line heating source in a semi-infinite slab of infinite depth—two-dimensional problem;
(b) circular ring heating source in a semi-infinite slab of infinite depth—two-dimensional problem;
(c) line heating source of finite length in a semi-infinite slab of infinite depth—three-dimensional problem;
(d) square line heating source in an infinite quadrant of infinite depth—two-dimensional problem.