Aerospace Science and Technology 58 (2016) 26–35

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Model to predict water droplet trajectories in the flow past an airfoil

S. Sor [a], A. García-Magariño [b], A. Velazquez [c,*]

[a] National Institute of Aerospace Technology (INTA), 28850 Madrid, Spain
[b] ISDEFE Engineering Systems for Spanish Defense Beatriz de Bobadilla 3, 28040 Madrid, Spain
[c] Aerospace Propulsion and Fluid Mechanics Department, Universidad Politecnica de Madrid, Plaza del Cardenal Cisneros 3, 28040 Madrid, Spain

ARTICLE INFO

Article history:
Received 8 January 2016
Received in revised form 28 July 2016
Accepted 29 July 2016
Available online 4 August 2016

Keywords:
Theoretical model
Droplet trajectory
Droplet deformation
Droplet drag
Experimental rotating arm
Airfoil

ABSTRACT

A theoretical model is presented to predict water droplet trajectories in the flow past an airfoil. The model considers droplet deformation and includes a drag coefficient that accounts for the influence of flow acceleration. This is because, as seen from the reference frame of the droplet, the flow accelerates as the airfoil approaches, even if the airfoil moves at constant velocity. To validate the theoretical model, a series of experimental tests have been carried out in a rotating arm facility. Three parameters were changed in the experiments: 1) the size of the model airfoil (radius of curvature 0.103 m, 0.070 m, and 0.030 m), 2) its velocity (50 m/s, 60 m/s, 70 m/s, 80 m/s, and 90 m/s), and 3) the droplets' initial diameters (in the range from 550 μm to 1050 μm). Comparison between the results obtained using the theoretical model and those collected in the experimental tests (droplet tracking was carried out using a high speed imaging system) showed a good agreement. This suggests that, within the range of parameters that has been tested, the proposed theoretical model could be confidently used for trajectory prediction purposes.

1. Introduction

The problem of predicting liquid droplet trajectories, inside a gas flow is of interest in many fields of science and engineering. They include, among many others, forensic applications [1], solvent extraction [2], electrostatic enhancement of liquid–liquid contacting processes [3], ink-jet printer design [4], spray modeling [5,6], and design of nuclear fusion subsystems [7].

In aeronautics, computation of water droplet trajectories is of interest, among others, for the purpose of simulating icing conditions. As compared to situations in other technical fields, droplet trajectories in these aeronautics-type conditions are characterized by the fact that, in the vicinity of an incoming airfoil, the flow, as seen from the reference frame of the droplet, accelerates with a non-constant acceleration. This is in contrast to other cases in which the flow is either steady or it accelerates with constant acceleration. In this context of dealing with non-constant velocity flows, the interested reader is directed to the work of Rendall and Allen [8] that developed a finite volume code in which droplet motion is tracked using mesh connectivity. This work is of interest because, instead of using a pure Lagrangian approximation, the authors couple the droplet motion to the finite volume computation of the surrounding flow in a computationally efficient way. Also of interest is the work by Saeed et al. [9] in which the air model of the code is of the panel method type, thereby allowing for a much faster computation of droplet trajectories, albeit at the expense of a smaller accuracy. On the other hand, a comprehensive work that involves both theoretical modeling and experimental testing has been reported by Papadakis et al. [10]. Additional experimental data in the field can be accessed in the study published by Reehorst and Ibrahim [11].

With regard to the basic Fluid Mechanics aspects of the modeling of droplet trajectories, it is worth mentioning, first, the review article by Aggarwal and Peng [12] published in 1995. In Section 1 of that article, the authors review a number of droplet dynamic models and conclude that flow non-uniformity and acceleration affect the aerodynamics forces critically. Also, they report a large uncertainty regarding their actual contribution to the total drag and lift forces. Even though it is not a review article by itself, reference [13] by Schmehl also contains a quite interesting discussion on droplet drag and dynamics models, including those models that deal with flow non-uniformity and acceleration. A method to reduce the number of similarity parameters needed to close a droplet trajectory model (under certain assumptions) has been published by Bragg [14]. In the case of unsteady Stokes flow, Maxey and Riley [15] have proposed a generalized equation of motion for a sphere in a non-uniform flow. In the case of an inviscid unsteady non-uniform flow, Auton et al. [16] have derived

* Corresponding author.
E-mail address: angel.velazquez@upm.es (A. Velazquez).

http://dx.doi.org/10.1016/j.ast.2016.07.015
1270-9638/

a general expression for the fluid force on the body. The case of drop deformation under steady conditions for a variety of fluids other than water has been extensively studied experimentally by Hsiang and Faeth [17]. Empirical correlations for the drag coefficient of gas bubbles inside a liquid (the opposite case to the one presented here) have been reported by Zhang et al. [18]. A statistical approach to droplet trajectory prediction during aerodynamics fragmentation has been developed by Flock et al. [19]. Regarding the fundamental aspects of droplet deformation that, in turn, affect droplet trajectory, the reader is directed to the excellent review presented by Theofanous [20]. In this review article, apart from making a critical review of the literature, the author proposes the existence of two fundamental droplet deformation modes that may explain the large variety of available experimental observations with regard to the shapes of the deformed droplets. From the standpoint of novel experimental techniques it is worth mentioning the work of Zarrabeitia et al. [21] that have recently developed a stereo reconstruction technique that allows for the recording of 3D droplet trajectories. Also, it is also important to refer to the work of Theofanous and Li [22] in which the authors present their laser-induced fluorescence technique.

Finally, and because of their relevance for the present work, it is important to discuss in some detail the studies presented by Temkin and Metha [23], Igra and Takayama [24], and Jourdan et al. [25]. The article by Temkin and Metha [23] presents an experimental study on the motion of water droplets inside accelerating and decelerating flows. The interesting modeling aspect of this study is that the authors assume a functional relationship between the drag coefficient and the so called acceleration parameter that is defined as the time derivative of the slip velocity divided by the square of the slip velocity itself. The experiments were carried out in a shock tube. Droplet diameters were in the range from 100 μm to 200 μm, which led to Weber numbers such that the authors assumed a negligible droplet deformation. Their conclusion (which is in contradiction with some other studies published in the literature) is that the unsteady drag of a sphere in decelerating flow is always larger than the steady drag at the same Reynolds number, while is it always smaller if the surrounding flow accelerates. Igra and Takayama [24] also used a shock tube facility but, in their case, with non-deformable spheres made up of polystyrene, nylon, and polyamide. Their diameter ranged from 0.5 mm to 4.8 mm. The Reynolds number covered in their experiments was in the range down from 6000 up to 100,000. Incident Mach numbers in the shock tube were 1.27, 1.50, and 1.80. In their conclusions, the authors reported unsteady drag values about 50% larger than the corresponding steady values in these shock tube conditions. More recently, Jourdan et al. [25] have presented another quite comprehensive experimental study based, also, on a shock tube type test rig. They used non-deformable spheres (made of either polystyrene or nylon) with diameters ranging from 500 μm to 6.6 mm. In their tests, the authors found that the unsteady drag is always larger than the steady drag at the same Reynolds number and explicitly stated at the end of their "Results and Discussion" section (section 4) that the acceleration parameter proposed by Temkin and Metha [23] may not be the relevant characteristic parameter for the flows that they considered. Even though these two experimental studies presented some differences that might affect the conclusions (droplet were deformable in reference [23] and non-deformable in reference [24], and velocities, and associated compressibility effects, were also somewhat different) the conclusion is that the field still is quite alive and that no definite conclusions are available so far.

The novelty of the work presented in this article consists of proposing a new theoretical model on droplet deformation and trajectory that is validated afterwards in a series of experimental tests in a rotating arm facility. The model, formulated as a set of three ordinary differential equations involves the presence of the so-called acceleration parameter (already proposed, and contested, also, by other researchers) and an equation for the deformation of the droplet. Then, the specific novelty aspects of the study are twofold: a) both the acceleration parameter and the droplet deformation equation enter simultaneously into the model (previous studies considered non-deformable droplets only), and b) because of this, although the functional form of the acceleration parameter is hypothesized, its actual parametric dependency needs to be characterized, and this is done via experimental testing.

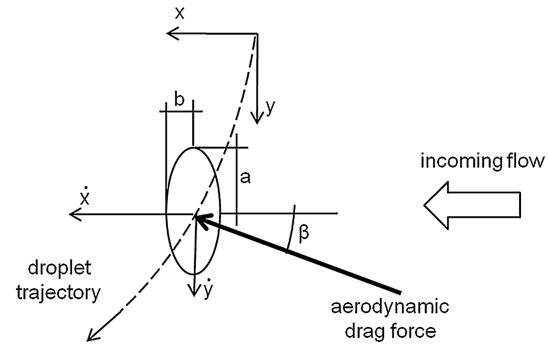

Fig. 1. Sketch of the problem under consideration.

Regarding the organization of the present article the theoretical model is presented in Section 2. The experimental rotating arm rig is described in Section 3. Model and experimental results are compared and discussed in Section 4 and, finally, conclusions are presented in Section 5.

2. Theoretical model

It is assumed that the droplet motion is governed by three equations: two dynamics equations (1)–(2) that represent the equilibrium of forces in the horizontal (x) and vertical (y) directions, and one equation (3) that models the droplet deformation. This equation (3) influences the droplet drag force because, indirectly, it allows for the computation of the droplet cross-section area normal to the incoming flow. Fig. 1 shows a sketch of both the acting forces and the axis of coordinates. The coordinates' axes (fixed in space) are located in the droplet centre of mass at the precise moment when it enters the measurement window.

Apart from some other considerations, the model presented hereafter is based on three main hypotheses that have been verified analyzing the experimental data obtained during the completion of the experimental campaigns. These hypotheses are:

- The "y" component of the incoming airflow $|V_{air\_y}|$ is very small ($V_{air\_y} \cong 0$). This assumption (that also implies that $|V_{air\_x}| \gg |V_{air\_y}|$) means that the model is valid, only, in the vicinity of the stagnation streamline (stagnation region) of the incoming airfoil.
- It is assumed that the droplet deforms as an oblate spheroid.
- Because of the first hypothesis, the slip velocity in the horizontal direction is much larger than the slip velocity in the vertical direction (this is, of course, not true during the initial instants of the droplet trajectory but velocities are very low and droplet deformation is negligible at these stages). Then, it is further hypothesized that the droplet deforms, only, along the vertical direction that is perpendicular to the direction of the much larger horizontal slip velocity. The practical implication of this third hypothesis is that forcing terms in the equation that models droplet deformation (equation (3)) depend, only, on the horizontal slip velocity; thereby decoupling, effectively, the "x" equation of motion (equation (1)) and the