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Free vibration of FG-CNT reinforced composite skew plates

Y. Kiani

Faculty of Engineering, Shahrekord University, Shahrekord, Iran

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ABSTRACT

Present study deals with the free vibration analysis of skew plates made from functionally graded carbon nanotube reinforced composites. Carbon nanotubes as reinforcements are distributed across the thickness of the plate. Distribution pattern may be uniform or functionally graded. The developed formulation from a Cartesian coordinate system is transformed to an oblique coordinate system to satisfy the boundary conditions. The virtual strain and kinetic energies of the plate are obtained using the first order shear deformation plate theory. Ritz method whose shape functions are developed according to the Gram–Schmidt process is implemented to construct an eigenvalue problem associated to the natural frequencies of the plate. The developed solution method is general and may be used for arbitrary boundary conditions of the plate. Results are compared for isotropic homogeneous and composite laminated plates in skew shape with the available data in the open literature. Afterwards numerical results are provided for skew plates reinforced with carbon nanotubes. It is shown that volume fraction of carbon nanotubes and their distribution pattern are both influential of natural frequencies of the carbon nanotube reinforced plates. Generally, the higher the volume fraction of carbon nanotubes, the higher the natural frequencies of the skew plate.

1. Introduction

Due to their exceptional mechanical properties, carbon nanotubes (CNTs) are known as an excellent candidate to reinforce the composites. CNTs have higher elasticity modulus in comparison to the polymeric and metallic matrices which results in a composite with enhanced stiffnesses. Therefore, volume fraction of CNTs is an important factor on structural response of composites reinforces with CNTs.

Another factor which may affect the global and local structural response of a composite media reinforced with CNTs is their distribution pattern. Distribution of CNTs in a matrix may be uniform or functionally graded according to a prescribed function. An overview of the available works on the mechanical and thermal properties of a composite media is provided by Liew et al. [1].

As shown by Shen [2], mechanical response of a rectangular plate may be enhanced with the introduction of a prescribed functionally graded pattern for the CNTs. Shen [2] exhibited that, bending moments of the plate may be alleviated significantly with the introduction of functionally graded CNTs where the top and bottom surfaces of the plate are enriched with the maximum volume fraction of CNT and the midsurface is free of CNT. Following this research, various investigators analysed the structural behaviour of CNT reinforced composites in various shapes. An overview of the available works on the vibration of plates made of functionally graded carbon nanotube reinforced composites (FG-CNTRC) is provided in the next paragraphs.

Based on the first order shear deformation plate theory and finite elements formulation, natural frequencies of an FG-CNTRC plate are obtained by Zhu et al. [3]. Free vibration of rectangular plates [4] and skew plates [5] are investigated by Zhang et al. using the element free methods. In two other studies, also, Zhang et al. [6,7] examined the free vibration characteristics of elastically restrained rectangular plates and plates which are simply supported in two opposite edges. For rectangular plates which are resting on elastic foundation, Zhang et al. [8] investigated the free vibration characteristics using an element free moving least squares Ritz formulation. For rectangular laminated plates with general boundary conditions and composed of FG-CNTRC layers, Lei et al. [9] investigated the free vibration characteristics using an element free method. Free vibration behaviour of FG-CNTRC plates in an arbitrary quadrilateral shape is investigated by Malekzadeh and Zarei [10] based on a two-dimensional generalised differential quadrature method. Malekzadeh and Heydarpour [11] developed a mixed solution method based on Navier and Layerwise formulations to investigate the free vibration characteristics of sandwich rectangular plates containing the FG-CNTRC layers. Due to the adoption of Navier solution method, plates which are simply supported all around may be analysed only. A higher order shear deformation

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E-mail addresses: y.kiani@aut.ac.ir, y.kiani@eng.sku.ac.ir.

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plate theory is used by Natarajan et al. [12] to investigate the free vibration of rectangular plates. Based on a two step perturbation technique, Wang and Shen [13,14] investigated the linear and nonlinear free vibration characteristics of FG-CNTRC plates and sandwich plates with FG-CNTRC face sheets. The provided solution method in these researches, may be used only for plates which are simply supported in flexure all around, while movable or immovable in normal to edges directions. Mirzaei and Kiani [15] applied the Chebyshev–Ritz method to the dynamic motion equations of the perforated plate to investigate the free vibration characteristics of moderately thick FG-CNTRC rectangular plate with a centric rectangular cut-out. Wang et al. [16] developed a semi-analytical solution to study the free vibration of thin rectangular FG-CNTRC plates using the classical plate theory formulation. Based on the conventional Ritz method accompanied with the Lagrangian multipliers technique, Kiani [17] investigated the free vibration characteristics of FG-CNTRC plates located on point supports. The solution method of this research is general and may be used for arbitrary number and position of point supports. Garcia-Macias et al. [18] analysed the free vibration and bending behaviour of FG-CNTRC skew plates. An efficient four-noded skew element with a total of twenty degrees of freedom is defined.

Similarly to free vibration behaviour of FG-CNTRC plates, forced vibration also has been the subject of many studies in the past years. However in comparison to free vibration, less attention is devoted to forced vibration. Dynamic response of rectangular plates made of FG-CNTRC plates subjected to dynamic loading is studied by Lei et al. [19] using a mesh free method. Wang and Shen [20] examined the geometrically nonlinear dynamic response of an FG-CNTRC rectangular plate under the action of lateral pressure. In this research, von-Kármán type of geometrical non-linearity is included into the formulation and plate is operating at various thermal environments. A two step perturbation technique suitable for plates with all edges simply supported is developed which may be used for both axially movable and immovable plates. Malekzadeh et al. [21] investigated the dynamic response of rectangular plate made of FG-CNTRC subjected to the action of a single moving mass. Finite element formulation is proposed to solve the motion equations of the plate suitable for arbitrary edge supports.

Above literature search reveals that, in comparison to rectangular plates, less attention is devoted to skew plates which is due to the more complex geometry. The present research examines the free vibration behaviour of moderately thick plates made of FG-CNTRC. First order shear deformation plate theory is used as the basic assumption to construct the kinetic and strain energies of the plate. The rectangular coordinate system is transformed to an oblique system which makes it easier to apply the boundary conditions of any type. Ritz minimization procedure is applied to the energies of the plate to establish the eigenvalue problem suitable for arbitrary in-plane and out-of-plane boundary conditions of the skew plate. To increase the convergence of the problem, the basis shape functions are approximated using the Gram–Schmidt orthogonal shape functions. The developed formulation is general and may be used for arbitrary combinations of boundary conditions. Numerical results of this study are compared with the available data in the open literature to assure the validity of the proposed formulation. Afterwards, parametric studies are given for FG-CNTRC skew plates.

2. Basic formulation

A skew plate with thickness h, edges a and b is considered. Orthogonal coordinate system is assigned to the corner of the mid-surface of the plate. The assigned coordinate system, geometrical characteristics and schematic of the plate are shown in Fig. 1.

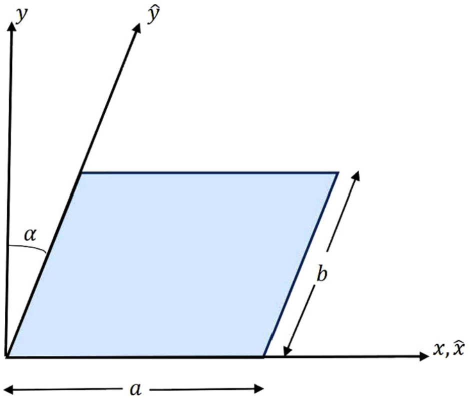

Fig. 1. Schematic, coordinate systems and geometrical characteristics of FG-CNTRC skew plate.

Table 1
Volume fraction of CNTs as a function of thickness coordinate for various cases of CNTs distribution [23–25].

CNTs distribution	V_{CN}
UD CNTRC	V^*_{CN}
FG-V CNTRC	$V^*_{CN}\left(1+2\frac{z}{h}\right)$
FG-O CNTRC	$2V^*_{CN}\left(1-2\frac{\lvert z\rvert}{h}\right)$
FG-X CNTRC	$4V^*_{CN}\frac{\lvert z\rvert}{h}$

Distribution of CNTs across the thickness of the plate may be uniform or functionally graded. When distribution of CNTs across the plate is functionally graded, it is usually referred to as functionally graded carbon nanotube reinforced composite (FG-CNTRC) skew plate. From the mathematical point of view, various dispersion profiles may be considered for the CNTs across the thickness of the plate, however, linearly graded patterns of CNTs are more observed in the researches due to their consistency with the fabrication processes [22]. As a result, three types of FG-CNTRC plates may be achieved which are known as FG-V, FG-X and FG-O. These three types along with the uniformly distributed (UD)-CNTRC skew plate are considered in the present research. Table 1 presents the distribution of volume fraction of CNT as a function of thickness coordinate in various CNTRC plates.

It is easy to check from Table 1 that, both uniform and functionally graded patterns of CNTRC plates will have the same total volume fraction of CNTs which is denoted by V^*_{CN}. Through such feature, the dynamic characteristics of UD- and FG-CNTRC may be compared with respect to each other. V^*_{CN} may be obtained as a function of mass density of CNTs, ρ^{CN}, mass density of matrix ρ^m and mass fraction of CNTs w^{CN} as

$$V^*_{CN} = \frac{w^{CN}}{w^{CN} + \rho^{CN}/\rho^m - w_{CN}\rho^{CN}/\rho^m} \tag{1}$$

Referring to Table 1 and comparing the distribution pattern of CNTs reveals that, in FG-X pattern, the top and bottom surfaces of the plate are enriched by the maximum volume fraction of CNTs whereas the mid-surface is free of CNTs. In FG-O, distribution pattern is inverse. The top and bottom surfaces are free of CNTs and the mid-surface is enriched with the maximum volume fraction of CNTs. In type FG-V, the bottom surface is free of CNT and the top one is enriched with the maximum volume fraction of CNT. In UD type, unlike the other four FG types, volume fraction of CNT is constant at each surface of the plate.

Various methods are proposed to estimate the effective material properties of the CNTRC media. Among them, Mori–Tanaka scheme