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Stress concentration at the corners of polygonal hole in finite plate

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ABSTRACT

A generalized formulation to determine the stresses around the polygonal shaped hole in anisotropic finite plate is presented in the paper. The stress concentration at the rounded corners of the polygonal hole in finite plate subjected to in-plane loading is derived using complex variable approach in conjunction with boundary collocation method. The influence of plate size, hole geometry and location, material anisotropy and loading conditions on the stress concentration around the polygonal hole is studied and presented in the paper. The results obtained through present method are validated by comparing with literature and finite element solutions.

1. Introduction

The polygonal shaped cutouts are provided in a plate like structural components for the requirement of service in many engineering applications like windows panels of aircraft, openings in marine vehicles, opening for fixtures in space vehicle, perforated plates, etc. which are generally made of laminated composite materials. The presence of the sharp corners of polygonal hole adversely affects the stress field in the plate when subjected to various loading. The material anisotropy and the size of plate also affect the stresses in the plate and raise the stress concentration around the hole. The high stress concentration may lead to the catastrophic failure of the components. To understand the catastrophe of the component it is necessary to estimate the stresses produced in the plate under different loading conditions.

Various methods have been used by different researchers to determine the stresses around the cutout in a plate. Chen [1] used special finite element method to obtain the stress concentration around hole in infinite plate. Wang et al. [2] used complex boundary integral method to obtain the stress field in an infinite plate with multiple circular hole. Muskhelishvili [3] has introduced a complex variable method to solve the problems of theory of elasticity. Savin [4], Lekhnitskii [5], Rao et al. [6], Sharma [7–9], Batista [10], Rezaeepazhand and Jafari [11–13], Daoust and Hoa [14], etc. have presented the analytical solution to estimate the stress concentration around various shapes of polygonal hole for isotropic and anisotropic plate subjected to remote loading using Muskhelishvili's [3] complex variable approach. The size of the plate is considered infinite compared to the size of hole in all these literatures. But in many practical cases the size of plate is finite for which these solutions are inappropriate.

The derivation of the stress field in finite plate with circular and elliptical hole is presented by Ogonowski [15], Madenci et al. [16], Xu et al. [17], Lin and Ko [18], Durelli et al. [19], etc. for anisotropic and isotropic materials. The solution of deriving the stresses around rectangular hole and regular polygonal hole in finite plate is presented by Pan et al. [20] and Jafari and Ardalani [21] respectively for isotropic material. For anisotropic finite plate, the solutions of stress concentration factor around rectangular hole and stress intensity factors at cusp of hypocycloidal hole are presented by Chauhan and Sharma [22,23]. All these solutions have utilized the boundary collocation method proposed by Bowie and Neal [24] and Newman [25]. The study of the existing literatures reveals that the solution of deriving stresses around regular polygonal hole in anisotropic finite plate has still not been attempted, within the best of author's knowledge.

An attempt is made here to derive a generalized formulation to obtain the stresses around the regular polygonal shaped hole in anisotropic finite plate under the action of in-plane loading. The stress functions are derived using complex variable approach in conjunction with boundary collocation method. The effect of plate size, hole geometry, hole orientation and location, material properties and loading conditions on the stresses around the hole is also studied.

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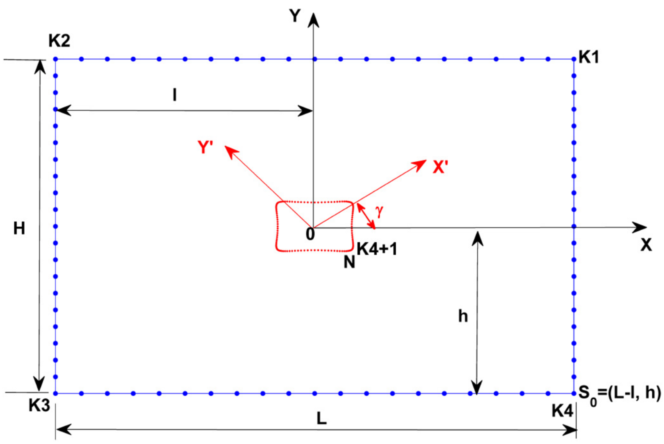

Fig. 1. Collocation points on the plate and hole boundary.

2. Analytical formulation

A thin rectangular anisotropic plate of length L and height H with a polygonal hole located at l and h distance from the bottom and left edge of the plate respectively is under the application of in-plane loading along X′0Y′ frame, as shown in Fig. 1. Assuming the plane stress condition, the stress components in an anisotropic plate [5] can be obtained as,

$$\sigma_x = 2\,\mathrm{Re}\left[\sum_{j=1}^{2}\mu_j^2\varphi_j'(z_j)\right],$$
$$\sigma_y = 2\,\mathrm{Re}\left[\sum_{j=1}^{2}\varphi_j'(z_j)\right], \tag{1}$$
$$\tau_{xy} = -2\,\mathrm{Re}\left[\sum_{j=1}^{2}\mu_j\varphi_j'(z_j)\right]$$

where μ_j are the constants of anisotropy obtained by applying generalized Hooke's law, Airy's stress functions and compatibility conditions to anisotropic plate, $\varphi_j'(z_j)$ are the first derivative of Muskhelishvili's complex stress functions $\varphi_j(z_j)$ and $z_j = x + \mu_j y$. To derive the stresses in a finite anisotropic plate, the complex stress functions are derived using Boundary Collocation Method as discussed in following section.

The solution begins with the generation of numbers of collocation points on the boundary of plate and hole as shown in Fig. 1, where $K1 = N1 + 1$, $K2 = K1 + N1$, $K3 = K2 + N1$, $K4 = K3 + N1$ and $N = K4 + N2$, $N1$ and $N2$ are number of points on the side of plate and boundary of the hole respectively. The x and y coordinates of each collocation points are obtained as follows:

$$\begin{aligned}
(x_s, y_s) &= (L - l, -h) \quad s = 1\\
&= \left(x_{s-1}, y_{s-1} + \frac{H}{K1-1}\right) \quad 2 \le s \le K1\\
&= \left(x_{s-1} + \frac{L}{K2-1}, y_{s-1}\right) \quad K1+1 \le s \le K2\\
&= \left(x_{s-1}, y_{s-1} - \frac{H}{K3-1}\right) \quad K2+1 \le s \le K3\\
&= \left(x_{s-1} - \frac{L}{K4-1}, y_{s-1}\right) \quad K3+1 \le s \le K4\\
&= \left(\mathrm{Re}(z_s), \mathrm{Im}(z_s)\right) \quad K4+1 \le s \le N
\end{aligned} \tag{2}$$

where Re and Im stands for Real and Imaginary part respectively and

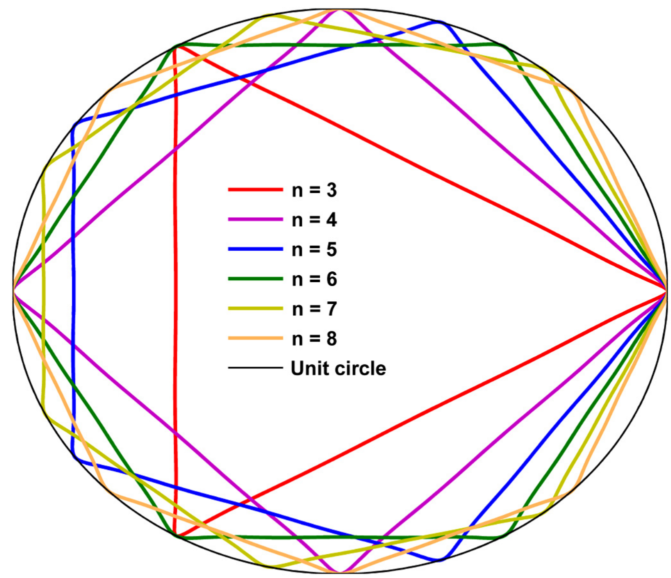

Fig. 2. Geometry of polygonal hole.

$$z_s = R\left[c_0\zeta_s + \sum_k \frac{c_k}{\zeta_s^{p_k}}\right] \tag{3}$$

where R is the size factor, $\zeta_s = e^{i\theta_s}$, $\theta_s = (s - K4 - 1) + 2\pi/N2$, c_0, c_k and p_k are the constants parameters to define different polygonal shape. For the polygonal hole of side n, $c_0 = 1$, $c_k = \frac{(\prod_{j=1}^{k}((j-1)n-2))}{n^k(1-kn)k!}$ and $p_k = kn - 1$. To inscribe the polygonal hole in to a unit circle $R = \frac{1}{1+\sum_k c_k}$ and to orient the polygon at angle α with respect to X axis, the constants c_k are multiplied with $e^{iq_k\alpha}$ where $q_k = p_k + 1$. Eq. (3) is the mapping function that maps the polygonal shape conformally on to a unit circular hole (see Fig. 2).

The geometry of the plate with hole in a complex plane, z-plane is obtained by $z_s = x_s + iy_s$. For the plate made of laminated composite material the plate geometry is defined in z_j-plane, due to affine transformation, as $z_{sj} = x_s + \mu_j y_s$. Considering the constants of anisotropy, the mapping function Eq. (3) takes form

$$z_{sj} = \frac{R}{2}\left[a_j\left(\frac{c_0}{\zeta_s} + \sum_k c_k\zeta_s^{p_k}\right) + b_j\left(c_0\zeta_s + \sum_k \frac{c_k}{\zeta_s^{p_k}}\right)\right] \tag{4}$$

where $a_j = 1 + i\mu_j$ and $b_j = 1 - i\mu_j$.

Multiplying Eq. (4) by ζ^K (K is the maximum power of ζ) and rearranging the terms,

$$\begin{aligned}
&Ra_j\sum_k c_k\zeta_{sj}^{p_k+K} + R(a_jc_0)\zeta_{sj}^{K+1} - 2z_{sj}\zeta_{sj}^{K} + R(b_jc_0)\zeta_{sj}^{K-1}\\
&\quad + Rb_j\sum_k c_k\zeta_{sj}^{K-p_k} = 0
\end{aligned} \tag{5}$$

Eq. (5) is polynomial equation of ζ that maps all the collocation points of z_j-plane on to ζ_j-plane. Out of $p_k + K$ number of roots of Eq. (5), the one that maps the polygonal shape to unit circular shape is selected and each z_{sj} is mapped to corresponding ζ_{sj} points. On each of this point in ζ_j-plane, the following force boundary conditions are imposed,

$$\pm F_x = \pm S_x(y_s - y_0) = 2\,\mathrm{Re}\sum_{j=1}^{2}\mu_j\left(\varphi_j(\zeta_{js}) - \varphi_j(\zeta_j^0)\right),$$
$$\mp F_y = \mp S_y(x_s - x_0) = 2\,\mathrm{Re}\sum_{j=1}^{2}\left(\varphi_j(\zeta_{js}) - \varphi_j(\zeta_j^0)\right) \tag{6}$$