Aerospace Science and Technology 55 (2016) 24–33

Contents lists available at ScienceDirect

Aerospace Science and Technology

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Aerodynamic shape optimization for minimum robust drag and lift reliability constraint

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ARTICLE INFO

Article history:
Received 8 November 2015
Received in revised form 23 April 2016
Accepted 4 May 2016
Available online 9 May 2016

Keywords:
Aerodynamic shape optimization
Robustness
Reliability
FORM
Sparse grid quadrature
Adjoint method

ABSTRACT

A methodology for shape optimization of aerodynamic bodies under uncertainties is presented. Flow-related and geometrical uncertainties are considered and quantified by probability distribution functions. The optimal shape is computed by minimizing a robust estimate of the drag coefficient subject to reliability constraint for the lift coefficient. The robust drag is formulated as a weighted sum of the mean and the standard deviation of the drag coefficient over the space of uncertain parameters. The mean and standard deviation of the drag coefficient are computed using sparse grid techniques. The lift reliability, defined by the probability the lift coefficient is lower than a reference value, is computed using First Order Reliability Method (FORM). A gradient-based optimization algorithm is used to obtain the optimal shape. The sensitivity derivatives of robust drag measure and the lift reliability with respect to the shape controlling and flow related design parameters as well as the uncertain parameters are computed using the adjoint problem for the flow. The methodology is applied to pure aerodynamic shape optimization, comparing optimal designs that arise from the formulation to optimal designs that correspond to special cases, including the case of no uncertainties. A 2D airfoil case is designed based on the Euler equations under uncertain Mach number and angle of attack and geometric variability.

1. Introduction

The availability of powerful Computational Fluid Dynamics (CFD) models has allowed the scientific community to investigate and develop a variety of algorithms applied to shape optimization, optimal active flow control with suction-blowing jets, topology optimization, etc. However, the resulting optimal design lacks good performance when the values of some parameters of the problem are uncertain or may vary within a range. Optimal designs based on a single value of the models parameters are very sensitive to uncertainties in the parameters in the sense that the performance deteriorates considerably in the neighborhood region where the parameters are likely to take values. Thus, the optimal design should take into account the variability or uncertainties of such parameters [51,50,46,45] by minimizing an overall measure of the performance over all possible values of the uncertain parameters and the sensitivity of performance to uncertainties. A multi-point optimization approach has been introduced to account for uncertainties by computing the performance in multiple points in the uncertain parameter space [34,16,23,35].

Probability distribution functions (PDFs) are often used to quantify uncertainties in simulations and probability calculus is applied to propagate the uncertainties in output quantities of interest (QoI). In design optimization the output QoI are associated with system performance measures involved in the objective function or the constraints. The mean and standard deviation are conveniently used as simple measures of uncertainty in QoI. Thus, the obvious choice in design optimization under uncertainties would be to minimize the mean value of the performance function and the standard deviation over the range of possible values of these uncertain parameters [39,41].

The mean and standard deviation are formulated as multi-dimensional integrals in the uncertain parameter space. The computation of these multidimensional integrals may be based on deterministic or stochastic approaches, including derivative-based, sampling and grid-based approaches. The derivative-based robust design uses a Taylor or asymptotic expansion and the multi-dimensional integrals are approximated by expressions that involve the first and second derivative of the performance variables with respect to the uncertain parameters [39,41,42,27,36]. Such approaches are quite fast, but lack accuracy in cases of large uncertainties, or in cases where the linearization of the performance function in the uncertain parameter space is not adequate such as in the case of transonic flow.

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http://dx.doi.org/10.1016/j.ast.2016.05.005
1270-9638/

Stochastic approaches for the estimation of the statistical moments are usually based on advanced Monte-Carlo (MC) methods [27,49,19] which are costly due to the very large number of analyses on the sample points required to compute these integrals. In addition, the sample estimates of the integrals are non-smooth functions of the design variables due to the variability of the samples between design points, complicating the use of gradient-based optimization algorithms over the uncertain parameter space. The grid-based approaches [8] are more accurate for the estimation of the uncertainties but they usually require a large number of CFD evaluations on the predefined grid nodes. The sparse-grid approach [47,14,5] is a remedy to the numerous required evaluations, substantially reducing the number of grid points and thus the computational cost in relation to grid-based and Gauss quadrature techniques. Using grid-based techniques, the multi-dimensional integrals for the mean and standard deviation become smooth functions of the design variables.

Uncertainties should also be taken into account for the estimation of the constraints involved in design optimization. The constraints should be satisfied for all possible values of the uncertain model parameters, an implementation that is not practical to carry out in most problems. Using PDF to quantify uncertainties and taking into account that the performance objective competes with the constraints, the requirement to satisfy the constraint at all possible values of the model parameters, even the values with relatively small plausibility based on the assigned PDF, results in significant deterioration of the system performance. In [45] for problems of shape optimization under uncertainty in CFD, the constraints are satisfied in the grid points used to approximate the multi-dimensional integrals involved in the robust performance measure. The drawback of this approach is that it fails to provide an overall measure of constrained violation over the parameter space. An alternative rational approach is to require that the constraint not to be violated with a given probability, formulating the constraint in terms of a reliability or, its complement, the probability of unacceptable performance. First-order reliability methods (FORM) [10,9] are most often used to approximate the resulting multi-dimensional reliability integrals over the domain in the uncertain parameter case where the performance criteria for the constraint are violated [28,30,21,13].

The optimal design under uncertainties is often formulated as a problem of minimizing a weighted average of the mean value and standard deviation of a performance function subject to the reliability constraints expressed in terms of the probability of unacceptable performance is lower than a small given probability value. In structural mechanics problems, the formulation has been applied for sizing, shape and topology optimization [30,20,17]. In CFD, the design optimization under uncertainties with reliability constraints may be found in the literature, almost exclusively for problems related with structural constraints in reliability-based aerostructural optimization problems. In [32], the wing mass and lift over drag ratio is minimized subject to probabilistic constraints on structural stresses using the FORM methodology. [11] applies a support vector machine method for the minimization of the probability of failure in stability (flutter) aeroelastic problems. The only flow related reliability-based shape optimization where the probability that the lift to drag ratio is lower than a given value, is computed and minimized may be found in [1], where the constraint is defined by the probability that the maximum stress exceeds another given threshold. The uncertain parameters are the angle of attack and the thickness of the wing plate and FORM is used to evaluate the reliability integral with respect to the two uncertain parameters.

This study presents a methodology for shape optimization of aerodynamic bodies by minimizing a robust measure of the drag coefficient under reliability constraint on the lift coefficient. The objective function is formulated as a weighted sum of the mean and the standard deviation of the drag coefficient. Minimizing the mean assures the smallest possible drag, while minimizing the standard deviation assures an optimal design corresponding to a drag value that is the least sensitive to parameter uncertainties. The sparse grid method is used to compute the mean and the standard deviation of the drag coefficient values at the grid points in the parameter space. The sparse grid approach based on different PDFs can be considered as a multipoint approach, although the sparse grid differs from the multipoint approaches [24] on the selection of the grid points to be consistent with the underline PDF and convenient to efficiently compute the statistical moments of the output quantities of interest. One of the contributions of this work is to impose the aerodynamic constraints on the lift coefficient in a probabilistic manner, requiring that the probability the lift coefficient is less than a reference value, denoted here as the probability of unacceptable performance, be bounded by a small user-defined probability. The FORM is used to estimate the probability of unacceptable performance or "failure" probability using the design point in the parameter space. A gradient-based approach is used to solve the constraint optimization problem. The adjoint approach for the underlining flow is applied for the computation of the first-order derivatives of the mean and the standard deviation as well as the probability of unacceptable performance with respect to the shape controlling parameters and the uncertain parameters, making the computation of sensitivities independent of the number of design variables and uncertain parameters.

The method is applied to the robust optimization of the shape of the RAE 2822 airfoil, the aerodynamic optimization of which has been presented in [45,22,6]. The shape is parameterized using Bézier control points [25]. The coordinates of these control points as well as the mean value of the angle of attack are considered as the design parameters to be optimized. The uncertainties considered are the values of the Mach number and the angle of attack as well as the geometrical uncertainties. Unlike existing geometry parameterization and uncertainty quantification schemes, in the present work the geometric variability is modeled by postulating PDFs to quantify uncertainties in the location of the control points. The proposed design optimization framework under uncertainties is illustrated for a 2D transonic flow governed by the Euler equations. The modeling using the Euler equations is for demonstration purposes, since the inviscid model does not always yield physically significant results [26]. The method of moving asymptotes (MMA) algorithm [48] is used to solve the constrained optimization problem. The importance of considering uncertainties, the effect of the type and magnitude of uncertainties, as well as the effect of the bound on the lift coefficient probability in the optimal design is investigated. The formulation for the robust aerodynamic optimization, minimizing a robust measure of the drag coefficient, under a reliability-based aerodynamic constraint imposed on the lift coefficient, constitutes the main contribution of this paper. The performance of the proposed robust drag optimization under lift reliability constraint is evaluated by comparing results with the ones obtained from the sole robust optimization, sole reliability-based optimization and deterministic optimization.

2. Aerodynamic shape optimization

2.1. Formulation of shape optimization neglecting uncertainties

In deterministic aerodynamic optimization the objective function to be minimized is usually the drag coefficient $C_D(\boldsymbol{\rho})$ constrained by the lift coefficient $C_L(\boldsymbol{\rho})$ not to exceed a predefined value, where $\boldsymbol{\rho} = (\boldsymbol{\rho}_g, \boldsymbol{\rho}_f)$ are the design variables, i.e. the geometrical variables $\boldsymbol{\rho}_g$ controlling the shape of the aerodynamic