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Pseudo-analytic approach to determine optimal conditions for maximizing altitude of sounding rocket

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ABSTRACT

A pseudo-analytic approach is suggested to determine the optimal launching conditions for maximizing the altitude of a sounding rocket flying with a constant mass flow of propellant in a standard atmosphere. The one-dimensional rocket momentum equation including thrust, gravitational force, and aerodynamic drag is considered, for which it is impossible to obtain the analytic solutions since the governing equation is nonlinear. The piecewise pseudo-analytic solutions are obtained with a constant control parameter introduced to make the velocity integral in the governing equation be analytic. The rocket flight in the standard atmosphere is analyzed by dividing the entire range into small intervals where the drag parameter or the gravitational acceleration can be treated as a constant in each interval. The pseudo-analytic approach gives precise predictions of the rocket velocity and the rocket altitude that agree well with the numerical ones. A characteristic equation exists and provides accurate predictions of the optimal mass flow rate for maximizing the altitude at burn-out state or at apogee.

1. Introduction

Many countries use sounding rocket programs in an effort to develop technologies related to sounding rockets, since scientific studies employing such programs are simple, efficient, and inexpensive compared to those with a satellite [1–12]. Most scientific measurements, observations, or experiments for sounding rocket missions are carried out near apogee. This is the case because the low speed near apogee provides unique opportunities to explore or observe the surrounding space in a short time period. Furthermore, there are some important regions of space that are too close to the earth's surface to be sampled by satellites; however, sounding rockets provide platforms for carrying out in-situ measurements in these regions [9]. Some microgravity environments [13,14] are carried after burn-out state and some scramjet experiments [15,16] are conducted during free-fall which provides a good hypersonic condition at a low cost. Therefore, the design target of a sounding rocket is the altitude at burn-out state or apogee. The rocket altitude can change based on the ejection conditions of the propellant jet. Therefore, it is necessary to determine an optimal condition for maximizing the altitude for given launching conditions.

The Goddard problem of optimal thrust programming for maximizing the altitude of a rocket in vertical flight has been extensively studied using variation methods, asymptotic approaches or optimal control theories [17–20]. These are not based on the analytic solution of the rocket momentum equation, since there is no general analytic solution due to the nonlinearity of the governing equation. There are also approximate solutions using the Taylor series expansion, the perturbation method or the least square method [21], but they are complex and do not provide information about the optimal conditions. An analytic exact solution of the rocket momentum equation including thrust, gravitational force and aerodynamic drag force exists only in a typical situation where the three forces are well balanced. A previous study [22] presented an analytic approach to obtain analytic solution and to determine the optimal conditions for the typical situations. This approach was extended to rocket flight in a standard atmosphere [23]. However the existence of an analytic solution requires the balance of the three forces and thus the typical control of the mass flow rate of propellant. Thus these analytic approaches have serious limitations in real applications. For instance, most sounding rockets use a constant mass flow rate of propellant in which the rocket motion cannot be solved with an analytic approach. Hence, in the present study, a pseudo-analytic approach to obtain an approximate solution and determine the optimal conditions for maximizing the altitude of a sounding rocket are suggested and verified.

We consider the motion of a sounding rocket launched in the vertical direction for simplicity. Then, the motion of a sounding rocket can be described using a one-dimensional momentum equation that includes thrust, gravitational force, and aerodynamic drag force. The rocket mass varies with time, and the aerodynamic drag

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Nomenclature

F	= thrust	p	= static pressure
G	= ratio between inertia and drag	T	= temperature
g	= gravitational acceleration	γ	= specific heats ratio
h	= altitude	ρ	= density
J	= pseudo drag parameter	Ω	= rocket mass ratio between total mass and dry-mass
K	= drag parameter	ω	= rocket mass ratio between adjacent intervals
M	= Mach number	*Subscripts*	
m	= rocket mass		
$\dot{m}$	= rate of rocket mass change or mass flow rate of propellant jet	a	= ambient air
		b	= burn-out state
q	= velocity parameter for rocket velocity	c	= rocket combustor
r	= control parameter for rocket velocity	e	= jet condition at rocket nozzle exit
t	= time	o	= ground state
u	= velocity of propellant jet	opt	= optimal condition for maximizing altitude
v	= rocket vertical velocity	s	= stationary state (apogee)

is proportional to the square of the rocket velocity, which makes the governing equation nonlinear. Thus, we cannot obtain an analytic solution in a general form. We also consider the case where the mass flow rate of propellant is constant for which analytic solutions of the rocket momentum equation do not exist. We cannot use the analytic approaches, but there is a possibility to extend the previous analytic approaches [22,23] to build a pseudo-analytic approach for finding solutions. The reason why we cannot obtain analytic solutions is that the governing differential equation cannot be integrated in an analytic way. However, if the governing equation is multiplied by a proper parameter, one side of the differential equation can be analytically integrated. On the other hand, the other side of the governing differential equation cannot be integrated in an analytic way. A similar situation occurs when we deal with rocket flight in the standard atmosphere [24], where the air density dramatically changes with the altitude. Further, the gravitational acceleration cannot be treated as a constant when rockets reach the upper atmosphere. Moreover, the aerodynamic drag coefficient changes with the flight Mach number especially around the Mach number of unity. Hence, the aerodynamic drag is a variable that changes with the altitude or rocket velocity. A previous study [23] shows that the "divide-and-conquer" strategy might be a way to avoid these serious issues. Hence, we can exploit this strategy to solve the problem in the present study. If we divide the entire flight range into intervals that are small enough, we can treat the following terms as constants in each interval: the parameter multiplied to both sides of the governing equation, the air density, the gravitational acceleration and the drag coefficient. We can then have piecewise pseudo-analytic solutions and also determine the optimal conditions at burn-out state or apogee.

The rocket model considered in the present study is the same one in the previous study [23] that is a simplified model based on the Korea Sounding Rocket Program (KSR II and III) [8]. KSR II is a solid propellant rocket with a total the weight of 2.0 ton, a diameter of 0.42 m and a length of 11.0 m. KSR III is a liquid propellant rocket with a weight of 6.1 ton, a diameter of 1.0 m and a length of 13.5 m. In the present study, we consider the medium specification between KSR II and KSR III.

In Section 2, the one-dimensional rocket equation in boost phase and coast phase are briefly described. Section 3.1 provides alternative approach to obtain a pseudo-analytic solution of the governing equation. Sections 3.2 and 3.3 show how to build the characteristic equations to obtain the optimal conditions for maximizing altitude at burn-out state and at apogee. Section 4 shows the procedure of the numerical discretization of the governing equation. Section 5 provides calculation conditions such as atmosphere, aerodynamic drag coefficient and launching conditions. Results of calculations are discussed in Section 6.

2. One-dimensional rocket equation

2.1. Governing equation in boost phase

The motion of a rocket in boost phase climbing in the vertical direction can be described with the following one-dimensional rocket equation including thrust and aerodynamic drag as follows [25,26]:

$$m\frac{dv}{dt} = F - mg - Kv^2, \tag{2.1a}$$

$$F = \dot{m}u_e + A_e(p_e - p_a), \tag{2.1b}$$

$$K = \frac{S}{2}C_d\rho_a. \tag{2.1c}$$

The mass flow rate $\dot{m}$ is equal to the rate of the rocket mass change and has a negative sign. In the present study, we consider the cases with a constant mass flow rate of propellant.

$$\dot{m} = \frac{dm}{dt} = const. \tag{2.2a}$$

The mass of a rocket decreases with the mass flow of propellant.

$$m = m_o + \int_o^t \dot{m}dt = m_o + \dot{m}t. \tag{2.2b}$$

If we use the constant specific heats ratio, the mass flow rate through a supersonic nozzle is determined as follows:

$$\dot{m} = \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}A_{th}\frac{p_c}{\sqrt{T_c}}. \tag{2.2c}$$

The subscript th denotes the throat of a rocket nozzle.

The term A_e in equation (2.1b) is the cross-sectional area at the nozzle exit. For an adiabatic nozzle flow, the total enthalpy is constant, and we can then assume that the jet velocity u_e and the pressure at exit are constant. However, the ambient pressure decreases with the altitude and thus the second term of the thrust increases with the altitude. The jet velocity has a negative sign since its direction is opposite of the rocket velocity; thus, the thrust term $\dot{m}u_e$ has a positive sign.