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Terminal height estimation using a Fading Gaussian Deterministic filter

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ABSTRACT

In a recent work by the authors the concept of Fading Gaussian Deterministic filter was investigated. The algorithm is based on a set of equations derived from the minimization of a cost function where earlier data are progressively de-weighted by a fading factor. In such a way, the estimation was proved to be less prone to problem unknowns. A tuning procedure was proposed that allows the resulting globally best estimator to evaluate the covariance of an effective measurement noise and the true estimation error, without any a-priori assumption. In the present paper, a general formulation is derived where the observed system is influenced by a control input. Also, a proof is derived for the proposed tuning criterion, which is shown to provide, under certain assumptions, the fading factor that best dampens the modeling errors with respect to measurement noise. The validity of the proposed approach is investigated by means of both numerical simulations and an experimental campaign, where height estimation is performed by fusing information from MEMS accelerometers and a barometric altimeter.

1. Introduction

This paper presents an estimation filter that features online tuning capabilities and its application to a data-fusion problem for height estimation. The algorithm is based on a recursive two-step set of equations known as Fading Gaussian Deterministic (FGD) filter, recently investigated by the authors in Ref. [1]. In such a framework, the filter was shown to be “optimal” because the gain matrix is computed through the formal minimization of a cost function, with no other assumption. In particular, earlier data are progressively de-weighted by a fading (or “forgetting”) factor, in order to make the estimation less prone to unknown external disturbances and/or model uncertainties and/or non-modeled dynamics, regardless of their nature (deterministic or stochastic). The introduction of the fading factor and the elimination of the so-called process noise covariance matrix $\boldsymbol{Q}$ make the difference with respect to a classical Kalman-like estimation process [2]. The approach was first developed by Norman Morrison in Ref. [3] and preliminarily elaborated in Ref. [4]. A similar concept was also investigated within the digital processing community in Refs. [5–7]. More recently, the Morrison filter was applied, with both non-recursive and recursive formulations, to a nonlinear tracking problem, where Levenberg–Marquardt methods were incorporated for improved convergence [8].

The main contribution provided in Ref. [1] consisted in a criterion for filter tuning, in order to minimize the estimation error, by selecting the best estimator in an ensemble of filters where only the fading factor value is varied. Once the filter is tuned, an effective measurement noise covariance $\boldsymbol{R}$ is directly estimated from the data (not as an a-priori assumption) and, as a by-product, the true covariance of the estimation error $\boldsymbol{P}$ is evaluated.

In the present paper, the recursive formulation introduced in Ref. [1] is extended to the general case where the system is influenced by a control input. In addition to the goodness-of-fit interpretation given in Ref. [1], a formal justification is also derived for the proposed tuning criterion, which is shown to provide the fading factor that best dampens the modeling errors with respect to the measurement noise. The effectiveness of the proposed approach and its ability to identify a globally optimum filter (within its own class) is investigated by means of both numerical simulations and an experimental campaign, where it is stressed that the proposed proof to the tuning technique allows quantifying the degree of knowledge of system dynamics. In particular, the algorithm is implemented in a test case relative to height estimation by fusing information from MEMS accelerometers and a barometric altimeter. A comparison is provided between the FGD algorithm and a simple method based on the so-called complementary filtering [9]. A simulation scenario is finally described where, in order to outline an evolving dynamic environment, the modeling accu-

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Nomenclature

$\boldsymbol{E}_k^-, \boldsymbol{E}_k$	computed predicted and corrected error matrices
h	count in time series
$\boldsymbol{H}_k$	observation matrix
$\boldsymbol{I}_q$	$q \times q$ unit matrix
$\boldsymbol{i}_k$	innovation vector
k	count in all time series
m	size of system state vector
$\boldsymbol{M}_k$	Morrison gain matrix
n	size of observation vector
$\boldsymbol{P}_k$	true covariance matrix of error $\boldsymbol{e}_k$
$\boldsymbol{P}_k^-$	true covariance matrix of error $\boldsymbol{e}_k^-$
$\tilde{\boldsymbol{P}}_k$	estimated covariance matrix of error $\boldsymbol{e}_k$
$\boldsymbol{Q}_k$	system noise covariance matrix
$\boldsymbol{R}, \boldsymbol{R}^*, \tilde{\boldsymbol{R}}$	true, assumed, and estimated measurement noise covariance matrices
$\boldsymbol{u}_k$	control input vector
$\boldsymbol{v}_k, \boldsymbol{w}_k$	true measurement noise and process noise vectors
$\boldsymbol{x}_k, \boldsymbol{x}_k^-, \tilde{\boldsymbol{x}}_k$	true state vector, predicted and corrected state vector estimates

Greek symbols

β	fading scalar factor
χ_k^2, ξ_k^2	tuning and scaling statistics
$\varepsilon_k, \tilde{\varepsilon}_k$	general and minimized cost functions
$\boldsymbol{\Phi}_k$	system state transition matrix
$\boldsymbol{\Gamma}_k$	control input model
ρ_k	scalar normalized squared residual
$\theta, \tilde{\theta}$	true and estimated scaling factors between $\boldsymbol{R}$ and $\boldsymbol{R}^*$

Subscripts and superscripts

T	tuning value

racy is artificially corrupted, with the consequent deterioration of the goodness-of-fit and the necessity to increase the filter's fading effect.

2. Basic specifications of the FGD filter

Suppose that, at time k, there is some true state vector $\boldsymbol{x}_k$ with dimension m which propagates approximately according to $\boldsymbol{x}_{k+1} = \boldsymbol{\Phi}_k \boldsymbol{x}_k + \boldsymbol{\Gamma}_k \boldsymbol{u}_k$, where $\boldsymbol{\Phi}_k$ is the state transition matrix, $\boldsymbol{u}_k$ is a known input vector with dimension l, and $\boldsymbol{\Gamma}_k$ is the control-input model. The observation $\boldsymbol{z}_k$ with dimension n approximately tracks this process as in $\boldsymbol{z}_k = \boldsymbol{H}_k \boldsymbol{x}_k$, where $\boldsymbol{H}_k$ is the observation matrix.

Given the time series of data $\boldsymbol{z}_0, \boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_k$, the aim is to compute estimates in a time series to the state vector, namely $\tilde{\boldsymbol{x}}_0, \tilde{\boldsymbol{x}}_1, \tilde{\boldsymbol{x}}_2, \ldots, \tilde{\boldsymbol{x}}_k$. Measurements are corrupted by a noise $\boldsymbol{v}_k$ with variance $\boldsymbol{R}$ [10], as in

$$\boldsymbol{z}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k \tag{1}$$

Let $\boldsymbol{R}^*$ be an assigned weighting matrix and $\beta \in (0, 1)$ be the fading factor. Consider the following cost function [3]:

$$\varepsilon_k = \sum_{h=0}^{k} \left(\boldsymbol{z}_{k-h} - \boldsymbol{H}_{k-h}\tilde{\boldsymbol{x}}_{k-h/k}\right)^T \boldsymbol{R}^{*-1} \left(\boldsymbol{z}_{k-h} - \boldsymbol{H}_{k-h}\tilde{\boldsymbol{x}}_{k-h/k}\right) \beta^h \tag{2}$$

where

$$\tilde{\boldsymbol{x}}_{k-h/k} = \begin{cases} \tilde{\boldsymbol{x}}_k & \text{if } h = 0 \\ \left(\prod_{j=1}^{h} \boldsymbol{\Phi}_{k-j}^{-1}\right) \tilde{\boldsymbol{x}}_k - \boldsymbol{\gamma}_{k-h/k} & \text{if } h > 0 \end{cases} \tag{3}$$

and

$$\boldsymbol{\gamma}_{k-h/k} = \sum_{j=0}^{h-1} \left(\prod_{m=0}^{j} \boldsymbol{\Phi}_{k-h+m}^{-1}\right) \boldsymbol{\Gamma}_{k-h+j} \boldsymbol{u}_{k-h+j} \tag{4}$$

provided Eq. (4) is defined for $h > 0$. The cost function defined by Eqs. (2), (3), and (4) is chosen to be minimized by the best choice of the estimated state vector, $\tilde{\boldsymbol{x}}_k$. Note that this last term appears in any term of the sum in Eq. (2) through $\tilde{\boldsymbol{x}}_{k-h/k}$, which is the back-projection of the current estimate, given all data and input history up to k. By minimizing the cost function, the best fit between the current estimate $\tilde{\boldsymbol{x}}_k$ and all previous available data is thus sought. Turning the estimation problem into a curve fitting exercise thus allows, thanks to elementary statistical analysis, the true error covariance $\boldsymbol{P}_k$ to be directly estimated from the data without prior knowledge of $\boldsymbol{R}$ (which is also estimated, as part of the process).

The least squares solution to $\tilde{\boldsymbol{x}}_k$ is found through differentiation of Eq. (2) with respect to $\tilde{\boldsymbol{x}}_k$. The derivation is reported in Appendix A for a time-invariant scenario. Note, however, that the time-variant case reported below can be obtained by a similar procedure, at the cost of a more cumbersome notation. Given $\boldsymbol{H}_k$, $\boldsymbol{\Phi}_k$, and β:

1. Input:

$$\boldsymbol{z}_k,\ \tilde{\boldsymbol{x}}_k^-,\ \boldsymbol{E}_k^-$$

2. Compute:

$$\tilde{\boldsymbol{x}}_k = \tilde{\boldsymbol{x}}_k^- + \boldsymbol{M}_k \left(\boldsymbol{z}_k - \boldsymbol{H}_k\tilde{\boldsymbol{x}}_k^-\right) \tag{5}$$

where
- *standard formulation*

$$\boldsymbol{M}_k = \boldsymbol{E}_k^- \boldsymbol{H}_k^T \left(\boldsymbol{H}_k \boldsymbol{E}_k^- \boldsymbol{H}_k^T + \boldsymbol{R}^*\right)^{-1} \tag{6}$$

$$\boldsymbol{E}_k = \left(\boldsymbol{I}_m - \boldsymbol{M}_k \boldsymbol{H}_k\right) \boldsymbol{E}_k^- \tag{7}$$

- *alternative formulation*

$$\boldsymbol{E}_k^{-1} = \boldsymbol{H}_k^T \boldsymbol{R}^{*-1} \boldsymbol{H}_k + \left(\boldsymbol{E}_k^-\right)^{-1} \tag{8}$$

$$\boldsymbol{M}_k = \boldsymbol{E}_k \boldsymbol{H}_k^T \boldsymbol{R}^{*-1} \tag{9}$$

3. Project ahead:

$$\tilde{\boldsymbol{x}}_{k+1}^- = \boldsymbol{\Phi}_k \tilde{\boldsymbol{x}}_k + \boldsymbol{\Gamma}_k \boldsymbol{u}_k \tag{10}$$

$$\boldsymbol{E}_{k+1}^- = \frac{\boldsymbol{\Phi}_k \boldsymbol{E}_k \boldsymbol{\Phi}_k^T}{\beta} \tag{11}$$

The above formulation matches the Kalman algorithm except for Eq. (11), where the fading factor β appears in place of the process noise matrix $\boldsymbol{Q}$. On the one hand, the effect of β was initially specified in Eq. (2) in order to model an exponential growth of the residuals when back-propagating the current estimate in the past. On the other hand, it is retrieved as an inflation coefficient in the forward propagation of the filter error covariance.