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Adaptive interacting particle system algorithm for aircraft conflict probability estimation

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ABSTRACT

Maintaining some specific security zones between aircraft to avoid collisions is mandatory in air traffic management. In this paper, we improve the accuracy of conflict probability estimation with an interacting particle system (IPS) algorithm. More precisely, a set of intermediate conflict zones is automatically created during IPS procedure to reduce the variability of conditional probability estimation. This method also increases significantly the convergence rate. The efficiency of this strategy is analysed on a simple air traffic scenario and compared with other IPS strategies.

1. Introduction

Maintaining a specific minimum separation distance between two aircraft to avoid collisions is mandatory in air traffic management (ATM). This safety rule is generally guaranteed by the air traffic control (ATC) that demands aircraft to fly at set levels or level bands, on defined routes or in certain directions. There are different ways of identifying aircraft position, such as radar (more accurate) and transponder data (less accurate). The corresponding uncertainty makes it difficult to evaluate conflict or collision probabilities between aircraft with accuracy. To describe the aircraft trajectory one could make use of the flight mechanic equations. A large number of parameters are involved, and stochastic processes are in fact often considered.

A simple usual way to estimate a probability is to consider straightforward Monte Carlo simulations. Nevertheless conflict probability between aircraft takes so low values that it makes straightforward Monte Carlo methods inapplicable if a sufficient accuracy is required. If we can predict conflict probability with less error, we could for example have aircraft fly closer together, and increase throughput. This might require that pilots manage their routes more precisely, and could get passengers home sooner and more safely. Thus, specific methods relative to rare event simulation have been proposed [1–3]. In simple cases, authors of [4,5] estimate the probability that a pair of aircraft being closer than a given distance with analytical or geometrical methods. It is shown in [6] that the IPS algorithm proposed in [1] is efficient when more complex air traffic scenarios and more advanced conflict zones have to be taken into account. An efficient IPS algorithm with importance sampling for rare switching diffusion can also be applied with hybrid stochastic processes [7]. This approach has been developed to estimate near-miss and collision probability in [8]. Moreover, some hierarchical IPS algorithms proposed in [9] are considered for free flight modelling described by advanced stochastic Petri nets [10]. Recently, [11] proposed to discard, in IPS algorithm, trajectories that are below some threshold given by air traffic complexity measures, which reduces the simulation time.

The IPS algorithm relies on the estimation of nested conditional probabilities. To optimise its performance, these probabilities should be sufficiently high and have the same order of magnitude. Taking this remark into account, we propose here an improvement to the IPS algorithm by considering an automatic creation of intermediate conflict zones using quantile estimation. This method enables us to reduce the relative errors on the estimated conflict probabilities, and increases the IPS convergence rate.

This paper is organised as follows. We first present the formalism for properly defining conflict zones. Then, we review briefly the principle of the different methods for rare event estimation such as the standard IPS algorithm. An original approach in order to make standard IPS algorithm implementation fully adaptive through a score function is then proposed in this article. It is the

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Table 1
Parameters for conflict zones.

Conflict type	Mid-term conflict	Short-term conflict	Conflict	Near-collision	Collision
k	1	2	3	4	5
d_k	4.5	4.5	4.5	1.25	0.054
h_k	900	900	900	500	131
Δ_k	8	2.5	0	0	0

main contribution of this article. We also specify the type of score function that should be used in the context of conflict probability estimation. Adaptive IPS algorithm is tested on a simple model of aircraft trajectory and compared with existing other methods in the final section.

2. Formalism for conflict zones

We review here the standard Markov chain based formalism for defining conflict zones. The position of an aircraft is assumed to belong to the space $\mathbb{R}^d$. Then, if one considers n_a aircraft, an air traffic scenario can be modelled by a Markov process $X_t = (X_t^1, \ldots, X_t^{n_a}) \in \mathbb{R}^{N_A}$, where $N_A = d \times n_a$. For a pair (i, j) of aircraft, $i, j \in \{1, \ldots, n_a\}$, one defines their relative speed and relative rate of climbing with $v^{i,j}(X_t)$ and $r^{i,j}(X_t)$.

Five types of conflict in air traffic management can be defined: medium-term conflict, short-term conflict, conflict, near-collision and collision [8]. Each of them is characterised with three parameters d_k, h_k and Δ_k, for $k = 1, \ldots, 5$. These parameters that are proposed in [8] and [11], are presented in Table 1. The parameters d_k and h_k define the diameter and the height of a cylinder. The Δ_k parameter is the time period over which the predicted positions are compared. Hence, a pair (i, j) of aircraft is said to be in conflict of level k [8] if the vector X_t belongs to the following subset of $\mathbb{R}^{N_A}$

$$D_k^{i,j} = \{x \in \mathbb{R}^{N_A},\ |y^{i,j}(x) + \Delta v^{i,j}(x)| \le d_k,\ \text{and}\ |z^{i,j}(x) + \Delta r^{i,j}(x)| \le h_k,\ \text{for some } \Delta \in [0, \Delta_k]\}. \quad (1)$$

In addition, aircraft i is said to be in conflict of a level k (with any other aircraft) [8] if the vector X_t belongs to

$$D_k^i = \bigcup_{j \neq i} D_k^{i,j}. \quad (2)$$

Finally, the whole scenario is said to be in conflict of a level k [8] if the vector X_t belongs to

$$D_k = \bigcup_{i=1}^{n_a} D_k^i. \quad (3)$$

In this article, one wants to estimate the quantities $\mathbb{P}\left(X_t \in D_k^{i,j}\right)$, $\mathbb{P}\left(X_t \in D_k^i\right)$ and $\mathbb{P}(X_t \in D_k)$ for a given air traffic scenario. Since these probabilities are rare, they cannot be estimated with Monte Carlo simulations. Specific algorithms have been proposed and are reviewed in the following section.

3. Review of rare event estimation methods for Markov processes

In this article, we are interested in estimating the probability that a Markov process $\{X_t,\ t \ge 0\}$ enters a set B before a final time T

$$\mathbb{P}(X_t \in B,\ \text{for some } t \le T). \quad (4)$$

In that case, the Markov process under consideration is assumed to have continuous state space. To estimate a rare event probability, we distinguish three different approaches: statistical, numerical and simulation techniques.

3.1. Statistical approach with the extreme value theory

Extreme value theory (EVT) focuses on the behaviour of the distribution tail of a real random variable, based on a reasonable number of observations [12,13]. This theory has been widely used to model extreme risks in meteorological phenomena [14], finance and insurance [12,15] and engineering [16] because of its generally applicable conditions. This theory is of great interest when one has to work with only a fixed set of data. Under some conditions, the founder theorem of the extreme value theory [17,18,12] claims that the maxima of an independent and identically distributed (i.i.d.) sample sequence converges to a generalised extreme value distribution (GEV). Nevertheless, studying the dynamic of Markov processes in the non-extreme regime (rare event probability set not reached) to deduce some conclusion on the rare event probability may lead to bad estimates, especially if the dynamic changes beyond the non-extreme regime.

3.2. Analytical results

Theoretical results for the approximation of (4) can be found in the literature and several cases have been successfully studied. For instance, the application of small set hitting probabilities for Gaussian and Rayleigh processes is proposed in [19,20]. Lévy processes, which frequently arise in financial engineering, are studied in [21]. Another example of rare events in continuous time can be found in [22], where the author presents a Poisson clumping heuristic. Such analytical approaches appear to be useful but require many simplifying assumptions on the underlying models. Moreover, due to complex analytical expressions involved by such techniques, only simple cases are workable. Moreover, the Poisson clumping heuristic does not rely on any theoretical justification.

3.3. Importance sampling

The idea of importance sampling is to sample the trajectory with another distribution for which the trajectories of the process are more likely to reach the rare event probability set before time T. An introduction is presented [23] and numerous examples are given in [24,25]. The works related to importance sampling in a dynamic framework mainly focus on finite or denumerable state space models. Some techniques are presented in [26–29] and different examples can be found in [30,31]. In the special case of finite time T, a review of existing methods and asymptotic analysis is presented in [32]. The choice of an efficient auxiliary distribution in the case of continuous state space requires the user to have a significant experience of the studied system, and may face delicate calculation problem.

3.4. The IPS algorithm

To increase the number of visits in the rare event probability set, the IPS algorithm also called splitting or importance splitting, relies on the fact that there exist some well identifiable intermediate regions before the rare event probability set. The trajectories that have reached an intermediate region before the final time T are multiplied, the others are killed. With such a method, the trajectories are somehow forced to reach the rare set before the final