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Semi-analytical approach for computing near-optimal low-thrust transfers to geosynchronous orbit

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ABSTRACT

In this paper, a novel semi-analytical approach is developed for solving minimum-time and minimum-fuel low-thrust transfers to geosynchronous orbit. The proposed method is mainly based on two intuitive control strategies, with one focusing on the instantaneous variation of orbit elements, and the other concerning the cumulative effect of thrust force. By optimizing the objective functions of the two strategies, analytical thrust-steering laws are derived for each case. With the use of a refined efficiency factor, thrust arc locations can also be optimized during the transfer. In addition, selection of weights and other parameters further improves the performance of the resulting trajectories. Finally, two examples of transfers are presented. The computed trajectories are very close to, or even better than the optimal results obtained from the traditional direct and indirect techniques. Due to its simplicity and good performance, the proposed method would be particularly useful for preliminary mission analysis.

1. Introduction

Since the first flight implementation of low-thrust electric propulsion (EP) in the 1960s, the innovative and enabling technology has been broadly accepted for operational applications in space missions. For example, geostationary telecommunication satellites routinely use EP system for station-keeping, which allows for a significant reduction of propellant mass. Planetary missions like NASA's Deep Space 1 and ESA's SMART-1 further demonstrate the feasibility of using low-thrust solar electric propulsion (SEP) for deep space exploration. Although the efficiency of low-thrust propulsion is highly appealing, its application to near-Earth transfers has long been problematic. Firstly, High-power EP system is needed to finish energetic Earth orbit transfers within specified duration. For SEP spacecraft with very low thrust-to-weight ratios, the resulting transfer involves a large number of revolutions enroute to the final desired orbit, which makes the trajectory design particularly challenging to solve. Multiple recent and ongoing developments have significantly increased the practicality for the use of EP for near-Earth orbit transfers [1]. As a result, a simple and efficient method for low-thrust trajectory optimization is of great value for preliminary mission design.

The problem of low-thrust many-revolution Earth orbit transfers has been studied for many years. Popular research topics involve computing minimum-time and minimum-fuel transfers from low Earth orbit (LEO) to geosynchronous orbit (GEO) and geosynchronous transfer orbit (GTO) to GEO. Early work by Edelbaum [2] used variational calculus to derive steering laws that model thrust as small perturbations to Keplerian orbital elements. Circi [3] studied the possibility of bringing the Artemis satellite onto GEO using only low-thrust propulsion and applied Pontryagin's principle to determine the minimum-time trajectory. SEPSPOT [4], a widely used program for computing optimal Earth-orbit transfers using SEP, solves the two-point boundary value problem (TPBVP) using a shooting method. However, the solution to the TPBVP is very sensitive to the initial guess for costate variables. Recently, Peng et al. [5,6] developed new symplectic methods for solving the TPBVP. The symplectic adaptive algorithm can not only possess high precision, but also preserve the symplectic structure of the nonlinear optimal control problem. In contrary to the traditional indirect technology, direct optimization methods generally exhibit a larger radius of convergence domain. Betts [7] described the collocation method to solve a 578-revolution transfer by a sparse nonlinear programming algorithm. Kluever and Oleson [8] developed a thrust-steering parameterization method for computing near-optimal Earth-orbit transfers and utilized the orbital averaging technique to relieve computation burden. Another popular direct optimization technique is the pseudospectral method. By varying the segment widths and polynomial degree, Darby et al. [9,10]

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presented a variable-order adaptive pseudospectral method to simultaneously improve the accuracy and computational efficiency. Besides the optimization methods, some work has also focused on using heuristic control laws to compute low-thrust orbit transfers. Kluever [11] and Petropoulos [12] developed blended control laws by investigating the nature of the variational equations for orbital elements. Ilgen [13] proposed a Lyapunov-optimal feedback control technique for transfers between orbits of different semi-major axis, eccentricity, and inclination. Naasz [14] presented three feedback control schemes based on different representations of the spacecraft state and gave a systematic approach for the selection of weighting functions. Later in [15], Petropoulos described another simple and well-developed feedback control algorithm, known as Q-law, and in a succeeding work by the author himself [16], the Q-law was refined to capture the complexity of various orbit transfers. The advantage of heuristics control laws lies in the speed of computation, while the drawback is that the solutions are generally non-optimal. Recently, researchers proposed hybrid approaches that combine the heuristic control law with a global optimizer [17–19]. The performance of the hybrid approaches was found to be as optimal as those of the direct and indirect methods.

While the traditional direct and indirect methods are clearly productive, they are mostly time-consuming, or even have difficulty of convergence in computing low-thrust many-revolution Earth orbit transfers, which make them expensive or unsuitable for preliminary mission analysis. With the aim of developing a simple and effective method for preliminary trajectory design, this paper presented a new semi-analytical approach to compute near-optimal low-thrust transfers to GEO. Based on two intuitive control strategies that focus on instantaneous variation of orbit elements and cumulative effect of thrust force, analytical steering laws are derived for the minimum-time transfer problem. By introducing the concept of "efficiency factor" which is different from the ones used in Q-law, minimum-fuel transfers with a mechanism for coasting can also be solved. In addition, with the help of a cooperative evolutionary algorithm for weight coefficients optimization, the quality of obtained solutions can be further improved. The proposed approach is capable to provide a reliable estimate of optimal transfers to GEO and would be particularly useful for preliminary mission analysis. Comparison with numerical simulations further confirms the validity of the proposed approach.

To this end, the remainder of this paper is organized as follows. In Section 2, a brief overview of low-thrust trajectory optimization is given, including the dynamic model and performance index. After discussing the low-thrust GEO acquisition requirement, two intuitive control strategies are introduced in Section 3, and Section 4 derives the analytical steering laws of the two control strategies. After that, numerical simulations are presented in Section 5, and finally, some conclusions and discussions are drawn in Section 6.

2. Low-thrust trajectory optimization

A general problem statement for low-thrust trajectory optimization can be stated as follows: determine the optimal control variable $\boldsymbol{u}(t)$, $0 \le t \le t_f$, that minimizes the performance index

$$J = J\big(\boldsymbol{x}(t_f), t_f\big) \tag{1}$$

subject to the equations of motion

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{t}, \boldsymbol{x}, \boldsymbol{u}) \tag{2}$$

and terminal state constraints

$$\boldsymbol{\phi}\big[\boldsymbol{x}(t_f), t_f\big] = \boldsymbol{0} \tag{3}$$

The state vector for the optimization problem is comprised of the classical orbital elements and spacecraft mass, $\boldsymbol{x} = [a, e, i, \Omega, \omega, M, m]^T$. The equations of motion are governed by the Gauss's form of the variational equations [20], plus the differential equation for mass-flow rate

$$\begin{aligned}
\frac{\mathrm{d}a}{\mathrm{d}t} &= \frac{2}{n\sqrt{1-e^2}}\big(1+2e\cos f + e^2\big)^{1/2} U \\
\frac{\mathrm{d}e}{\mathrm{d}t} &= \frac{\sqrt{1-e^2}}{na}\big(1+2e\cos f + e^2\big)^{-1/2} \\
&\quad \times \big[2(\cos f + e)U - \sqrt{1-e^2}\sin E N\big] \\
\frac{\mathrm{d}i}{\mathrm{d}t} &= \frac{r\cos u}{na^2\sqrt{1-e^2}} W \\
\frac{\mathrm{d}\Omega}{\mathrm{d}t} &= \frac{r\sin u}{na^2\sqrt{1-e^2}\sin i} W \\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \frac{\sqrt{1-e^2}}{nae}\big(1+2e\cos f + e^2\big)^{-1/2} \\
&\quad \times \big[2\sin f U + (\cos E + e)N\big] - \cos i \frac{\mathrm{d}\Omega}{\mathrm{d}t} \\
\frac{\mathrm{d}M}{\mathrm{d}t} &= n - \frac{1-e^2}{nae}\big(1+2e\cos f + e^2\big)^{-1/2} \\
&\quad \times \left[\left(2\sin f + \frac{2e^2}{\sqrt{1-e^2}}\sin E\right)U + (\cos E - e)N\right] \\
\frac{\mathrm{d}m}{\mathrm{d}t} &= -\frac{\|\boldsymbol{u}\|}{I_{sp} g_0}
\end{aligned} \tag{4}$$

where n, f, E are the orbit mean motion, true anomaly and eccentric anomaly, respectively; I_{sp} is the specific impulse; g_0 is the gravitational acceleration at sea level. $[U, N, W]^T$ represent acceleration components due to low-thrust propulsive force ($\boldsymbol{u}/m$) and orbital perturbation effects ($\boldsymbol{f}_p$)

$$\begin{aligned}
U &= U_1 + U_2 = \frac{\boldsymbol{u}}{m} \cdot \boldsymbol{e}_U + \boldsymbol{f}_p \cdot \boldsymbol{e}_U \\
N &= N_1 + N_2 = \frac{\boldsymbol{u}}{m} \cdot \boldsymbol{e}_N + \boldsymbol{f}_p \cdot \boldsymbol{e}_N \\
W &= W_1 + W_2 = \frac{\boldsymbol{u}}{m} \cdot \boldsymbol{e}_W + \boldsymbol{f}_p \cdot \boldsymbol{e}_W
\end{aligned} \tag{5}$$

Here we use an orthogonal tangential-normal (UNW) coordinate frame, where the unit vectors are defined by the position and velocity vectors of the spacecraft as

$$\boldsymbol{e}_U = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}, \qquad \boldsymbol{e}_W = \frac{\boldsymbol{r}\times\boldsymbol{v}}{\|\boldsymbol{r}\times\boldsymbol{v}\|}, \qquad \boldsymbol{e}_N = \boldsymbol{e}_W \times \boldsymbol{e}_U \tag{6}$$

The objectives of low-thrust trajectory optimization generally include two classes of performance index. One is the minimum transfer time

$$J_1 = t_f \tag{7}$$

the other is the minimum-fuel consumption for a fixed time transfer

$$J_2 = \frac{1}{I_{sp} g_0} \int_{t_0}^{t_f} \big\|\boldsymbol{u}(t)\big\| \mathrm{d}t \tag{8}$$

According to Pontryagin's Minimum Principle [21], the minimum-time problem leads to a constant maximum thrust modulus during the flight,

$$\min J_1 \Rightarrow \|\boldsymbol{u}\| \equiv T_{\max} \tag{9}$$

while the optimal control law for the minimum-fuel problem is derived as a bang-bang control, in which the thruster is turned on or off depending on the value of a switch function