Aerospace Science and Technology 56 (2016) 100–111

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

An applicable formula for elastic buckling of rectangular plates under biaxial and shear loads

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A R T I C L E I N F O

Article history:
Received 12 May 2016
Received in revised form 1 July 2016
Accepted 12 July 2016
Available online 16 July 2016

Keywords:
Elastic buckling
Rectangular plate
Simply supported
Rayleigh–Ritz method
Biaxial/shear loads

A B S T R A C T

As thin plates have relatively big thickness ratios, their elastic buckling usually occurs before the yielding. From beginning of the previous century, many researchers have considered various in-plane loading states on thin plates and have strived to find simple equations to predict the buckling load. However, there are few valid equations with negligible errors for a thin plate, when it is under all of in-plane loads. In this paper, using energy method, an applicable formula is suggested for a simply supported rectangular plate, which is under biaxial and shear loads. The biaxial loads can be applied in the compressive/compressive, compressive/tensile, and tensile/tensile states on the plate. Generally, 15 129 examples are considered for this problem. The aspect ratio of plates varies from 1 to 5 and for each case and with the known load ratios, the plate buckling coefficient is calculated. Then, by using the regression techniques and interpolation, it is tried to estimate a simple equation with minimum error to predict the buckling load. The confirmed results show that for the biaxial compression and shear state, the maximum error is 8% and for the compression–tension–shear and biaxial tension and shear states, it increases until 20%.

1. Introduction

Thin-plated structures are widely used in various engineering industries such as building, bridge, aerospace, marine, shipbuilding and so on. Thin plates usually have thickness ratio between 10 and 100 and in practical purposes they mostly buckle under in-plane axial and shear loading before yielding. Because they have the post-buckling behavior, prediction of the buckling load by an applicable equation with minimum error is very important for such structures.

In many years, the valuable efforts have been performed to find concise equations for the buckling load of flat plates under the various loading types and boundary conditions [1–3]. There are several methods to predict buckling loads of such plates. The older methods have been applied from near the end of the 19th century [1] that mostly included the method of integration of the differential equation and also, the energy method. Recently, the numerical methods have been considered as useful tools for the complicated problems. Generally, the exact solutions can be developed, when the plate is under uniformly distributed compressive in one direction or two perpendicular directions. For the latter state, Lebove [4] showed that one of half-waves in the buckled plate is always unit; but the other one can be achieved by an explicit solution.

In this way, numerous researchers have investigated other states of loadings and boundary conditions through the years. Using energy method, van der Neut [5] obtained the buckling load of a simply supported plate under a half-sine load distribution on the opposite sides and later Benoy [6] investigated this problem for a parabolic distribution. He considered four boundary conditions of plate: (i) ends and sides simply supported, (ii) ends clamped, sides SS, (iii) ends SS, sides C, and (iv) ends and sides C. Also, the loading was expressed in terms of the stresses at the panel edges and center. Benoy compared the obtained results with those of van der Neut. Later, Bert et al. [7] claimed that two previous works were based on an incorrect in-plane stress distribution. They used Galerkin solution to remove the existing deficiencies in the previous works, especially for a sinusoidal stress distribution and then, achieved more accurate results for the buckling load. They concluded that their analysis shows the buckling loads at higher plate aspect ratio increase relative to those obtained in the literature.

Bank and Yin [8] considered buckling of an orthotropic plate, simply supported on its loaded edges and free and rotationally restrained on its unloaded edges. Uniform uniaxial compression was applied on the loaded edges and the method of integration of the differential equation (exact solution) for the deflected plate was used. In this study, the effect of orthotropic properties of the plate material, the plate aspect ratio, the rotational restraint of the one

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http://dx.doi.org/10.1016/j.ast.2016.07.005
1270-9638/

loaded edge and the buckle half-wavelength was discussed. They showed that in the case of a plate with a free edge, the Poisson ratio appears explicitly in the boundary conditions. Finally, the buckling curves were presented for the results of parametric studies as well as typical composite materials.

Kang and Leissa [9] presented an exact solution for the buckling and free vibration of rectangular plates having two opposite edges simply supported, each subjected to an in-plane moment, with the other two edges being free. The exact solution was applied in term of an infinite power series, so that sufficient number of terms of the series must be taken to obtain accurate numerical results. The results showed that the critical buckling moment always occurs for a mode having one half-wave in the direction of loading and also, the buckling and frequency parameters depend upon the Poisson ratio. Furthermore, the used approach may be applied equally well to plates having other continuous boundary condition along their unloaded edges.

Elangovan and Prinsze [10] arranged a finite element shear buckling analysis with NASTRAN for flat rectangular plates with two free opposite edges and the other two edges with different boundary conditions. In some curves, the shear buckling coefficient which obtained for the boundary conditions was compared and emphasized that the in-plane flexibility of the supports is an important parameter in the structural design.

In recent decades, the numerical methods have been extended to increase the efficiency and ability. Sherbourne and Pandey [11] used differential quadrature method (DQM) for solving directly the partial differential equation governing the problem with prescribed boundary conditions. This method suggests polynomial approximations of partial derivatives of a function. They employed DQM to compare some examples and results with available standard solutions. Their experience showed that compactness and computational economy of the DQ model are praiseworthy. Later, Civalek [12] compared the methods of differential quadrature (DQ) and harmonic differential quadrature (HDQ). He used these methods for various analysis of thin isotropic plates and columns. Unlike DQ that uses the polynomial functions, HDQ uses harmonic or trigonometric functions as the test functions. Civalek applied both of methods on some examples such as elastic columns, circular, rectangular, skew, trapezoidal, eccentric sectorial, and square plates. He concluded that in the numerical examples, the results obtained with HDQ method are more accurate than the values calculated by using finite elements and finite differences and needs less grid points than the DQ method.

Liew et al. [13] formulated the radial point interpolation method (RPIM) for the buckling analysis of non-uniformly loaded thick plate. The RPIM is a mesh-free method, so that the problem domain is not divided into sub-domain to approximate the displacement (unlike the FEM). The buckling loads of the circular, trapezoidal and skew plates were calculated and compared with FEM. Furthermore, Civalek et al. [14] used discrete singular convolution (DSC) for buckling and free vibration analyses of rectangular plates subjected various in-plane compressive loads and with different boundary conditions. The mathematical foundation of this method is the theory of distributions and wavelet analysis. The obtained results were compared with those of other numerical methods.

Beyond the described investigations, many studies can be found that have been presented for buckling of thin plates under combinations of in-plane loads and various boundary conditions. Using energy method, McKenzie [15] gave an analysis of the buckling of a rectangular plate of arbitrary aspect ratio under combination of biaxial compression, bending and shear. In this investigation, the pair of sides of the plate to which bending is applied are assumed to be simply supported, while the other two sides are supported by edges members of arbitrary torsional and flexural stiffnesses. McKenzie generated some interaction curves for different aspect ratios and load ratios.

Liu and Pavlovic [16] broke-down external loads (direct, shear and bending loads) into four parts in the symmetrical and antisymmetrical forms. For a simply supported rectangular plate and using principle of super position, the Ritz energy technique was used to compute the buckling coefficient of the plate. They emphasized that the proposed approach based on formal plane stress elasticity solution enables the true distribution in any plate to be obtain irrespective of the complexity and/or arbitrariness of applied forced on any edges.

However, some equations have been approximately developed among pure shear, pure bending, combined shear and longitudinal compression, shear and bending load [17–19]. Although a few investigations can be found for the buckling behavior of plates under biaxial and shear loads, Wagner [20–22] established two formulas to calculate the critical shear stress of simply supported and clamped plates with given values of biaxial stresses:

$$\left(\frac{\tau_{crm}}{\tau_0}\right)^2 = \left(2\sqrt{1-\frac{\sigma_y}{\tau_0}}+2-\frac{\sigma_x}{\tau_0}\right)\left(2\sqrt{1-\frac{\sigma_y}{\tau_0}}+6-\frac{\sigma_x}{\tau_0}\right);\quad \text{all edges simply supported}$$

$$\left(\frac{\tau_{crm}}{\tau_0}\right)^2 = \left(2.31\sqrt{4-\frac{\sigma_y}{\tau_0}}+\frac{4}{3}-\frac{\sigma_x}{\tau_0}\right)\left(2.31\sqrt{4-\frac{\sigma_y}{\tau_0}}+8-\frac{\sigma_x}{\tau_0}\right);\quad \text{all edges clamped} \tag{1}$$

where

$$\tau_0 = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2$$

In above equation, σ_x and σ_y are axial stresses in x- and y-directions respectively. They have negative values when are tensile. To use this equation, the plate aspect ratio must be very large [22]. As a result, Eqs. (1) could not be used for usual aspect ratio of plates ($1 < \alpha < 5$).

Chen et al. [23] estimated a concise formula for the critical buckling stresses of an elastic plate under biaxial compression and shear (Eq. (2)). They considered the plate aspect ratio between 1 and 5.

$$\frac{\sigma_x}{\sigma_{x,cr}} + \left(\frac{\sigma_y}{\sigma_{y,cr}}\right)^{\gamma} + \left(\frac{\tau_{crm}}{\tau_{cr}}\right)^2 = 1 \tag{2}$$

where

$$\gamma = \begin{cases} 1; & 1 \le \alpha \le \sqrt{2} \\ \alpha^{[1-(\frac{\tau_{crm}}{\tau_{cr}})^2]}; & \alpha > \sqrt{2} \end{cases};\quad \alpha = \frac{a}{b}$$

In Eq. (2), σ_x is compressive stress in x-direction; $\sigma_{x,cr}$ is uniaxial compressive buckling stress in x-direction; σ_y is compressive stress in y-direction; $\sigma_{y,cr}$ is uniaxial compressive buckling stress in y-direction; τ_{crm} is modified shear buckling stress of the plate and τ_{cr} is pure shear buckling stress of the plate.

Chen et al. emphasized that the maximum error of the critical stress relationship in above equation is found to be less than 0.5% for $1 \le \alpha < \sqrt{2}$, 5% for $\sqrt{2} \le \alpha < 2$, and 10% for $2 \le \alpha < 5$ [23]. Eq. (2) shows that for $\alpha > \sqrt{2}$, without shear load ($\tau_{crm} = 0$), $\gamma = \alpha$. As a result, Eq. (2) is converted to $\frac{\sigma_x}{\sigma_{x,cr}} + (\frac{\sigma_y}{\sigma_{y,cr}})^{\alpha} = 1$. It can be shown that for the biaxial loaded plates, power of both of the terms must be unit [1,22], whereas here $\alpha > \sqrt{2}$.

In addition, according to Von-Mises criteria, DNV-RP-C201 has an equation which can be used to obtain inelastic buckling of unstiffened plate under biaxial compression and shear loads [24].