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Nonlinear dynamic analysis of the interaction between a two-dimensional rubberlike membrane and a liquid in a rectangular tank

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A B S T R A C T

In the aerospace industry, a rubberlike membrane and liquid interaction like in a rubber bag formed into a solid container has been used for the three axis control of satellites. This paper considers the two dimensional rectangular tank filled with a liquid of which surface is covered by a rubber membrane. The membrane is assumed to be the rubberlike membrane which includes material and geometrical nonlinearities. To overcome the moving boundary problem due to large deformations, an Arbitrary Lagrangian–Eulerian (ALE) description was used to define the moving boundaries. The interaction problem in a rectangular tank was evaluated by comparing it with the sloshing model and previous theories. The results present the importance of the nonlinearity arising from the membrane itself. Nevertheless, the nonlinearity of a liquid still plays an important role in larger amplitude oscillations.

1. Introduction

On the ground, sloshing means any motion of a free liquid surface inside a container. The dynamics of a liquid sloshing is physical phenomenon that is attractive for research. Stability and structural integrity of externally excited filled liquid storage tanks, moving containers, and the many associated applications have been the focus of a wide range of technologies and engineering. The literature reported a variety of analytical and numerical techniques for formulating slosh models for different practical geometries such as large-capacity liquid containers, propellant storage tanks or containers in airplanes, missiles, space vehicles, satellites, or space stations [1]. In the aerospace industry, an overview of the relevant physics and modeling techniques for sloshing liquids in space was given by Vreeburg and Veldman [2]. Sloshing experiments have been designed to provide more information on these issues [3,4]. Most of the reports, however, were concerned with rigid tanks. For example, Faltinsen et al. [5] derived a linear analytical solution for a liquid sloshing in a horizontally excited 2-D rectangular tank, which has been widely used in the validation of numerical models. It is shown that the theory is not valid when the water depth (h) becomes small relative to the tank width (l). This is due to secondary parametric resonance. It is then necessary to include nonlinearly interacting modes having the same order of magnitude. This is demonstrated for a tank with $h/l = 0.173$. They observed fluctuations of the excitation frequency in an initial period up to approximately 10 s. This effect was important to include in the theoretical model. There is good agreement with experimental free surface elevation when $h/l > 0.28$. Faltinsen and Timokha [6] developed a multimodal approach to describe the nonlinear sloshing in a rectangular tank with a finite water depth. Since the difference between natural frequencies decreases with decreasing liquid depth, this is more likely to occur at low liquid depths. This is a reason why the single dominant mode theories are invalid for small liquid depth. In addition, if the excitation amplitude increases, the liquid response becomes large in an increased frequency domain around the primary natural frequency. The method has been validated by comparing with model tests. Adaptive procedures have been established for all excitation periods as long as the mean liquid depth is larger than 0.24 times the tank width. When $h/l < 0:24$ and depth is not shallow, good agreement with experiments has been achieved for isolated excitation periods. Celebi and Akyıldız [7] described a liquid sloshing in two-dimensional tanks using the finite difference method. It is noted that the structural flexibility and free surface sloshing effects were not properly addressed in the previous studies.

In modern technology, structures have trended toward being designed thinner and lighter because of high flexibility of these structures. Moreover, researchers often find strong interactions of propellants with the elastic structure in the control system. For in-

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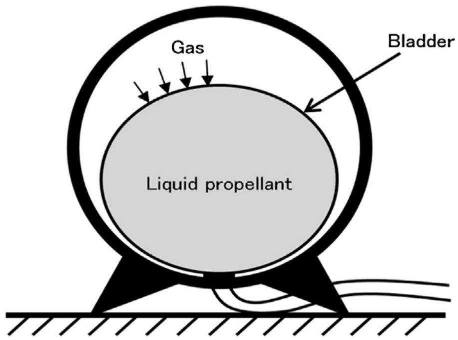

Fig. 1. Flexible bladder tank.

stance, the interaction of a rubberlike membrane and liquid could occur in the rubber bag of the solid container used for the three axis control of a satellite [8]. A flexible bladder tank for liquid has been used to push away the propellant during thrusting or to prevent the propellant from moving, as shown in Fig. 1.

Kawakami [8] studied a model of a liquid-filled cylindrical tank covered by a rubberlike membrane. The effects of the finite deformation of the membrane were examined using linear analysis and nonlinear finite element analysis. However, the moving boundary of the rubberlike membrane and liquid interaction causing by large deformations of the liquid region was neglected.

Bauer [9] studied the resonant frequencies of an incompressible and non-viscous liquid in a rigid cylindrical container, in which the free liquid surface was covered by a membrane or elastic plate. The assumption of small deformations was adopted to allow use of Fourier–Bessel series. This paper stated that a free liquid covered by a membrane or an elastic thin plate increased the resulting frequencies. After this study, Bauer and Chiba [10] extended the study in Ref. [9] to a structure filled with an incompressible viscous liquid, while Bauer and Komatsu [11,12] studied the coupled hydroelastic frequencies of an inviscid liquid in a circular cylindrical container, in which the free liquid surface was partially covered by an elastic annular plate. Bohun and Trotsenko [13] developed an analytic model solution for coupled free hydroelastic oscillations in an upright circular cylindrical container with the unperturbed free surface of the liquid being covered by an elastic membrane or plate. Gavrilyuk et al. [14] studied a circular membrane clamped to the edge of a rigid upright circular cylindrical tank filled with an incompressible liquid. This paper examined a linear problem that describes small relative coupled oscillations. The problem was defined to an operator differential equation completed with initial conditions, which imply initial variation and velocities of the stretched membrane. The Cauchy problem was reduced to a spectral boundary problem on linear natural modes (eigenfunctions). The numerical results demonstrated the efficiency of the proposed approximate method. The fast convergence to the solution is facilitated by a functional basis of a specific singular structure.

Similar treatments have also been performed on a rectangular container [15–17]. Bauer [15] studied the interaction of an elastic bottom with the free surface and liquid in a rigid tank that has a free liquid covered by a membrane in a rectangular container. The results showed that the free liquid surface was completely covered by an elastic structure; also the natural frequencies of the liquid and structure system increased because of the tension or stiffness of the covering membrane. Bauer [16] theoretically investigated the same issue and discussed the influences of the system parameters on the coupled natural frequencies. He mentioned that a free liquid surface was covered by a membrane in a rectangular container under large deformation. The liquid reduced the influence on the system, while the membrane increased the influence on the system. Specifically, the vibration was dampened due to the increasing membrane tension. Ikeda and Nakagawa [18] and Ikeda et al. [19] considered the nonlinear interaction of a liquid sloshing in rectangular and cylindrical tanks with an elastic structure in which the motion was orthogonal to the tank vertical walls. For a vertical sinusoidal excitation of an elastic structure carrying a rigid rectangular tank, Ikeda [20] determined the responses of the coupled systems when the structure natural frequency was approximately twice the liquid sloshing frequency. However, hydrodynamic systems, in which the free liquid surface was only partially covered with an elastic membrane, were evaluated. In most cases, the liquid is assumed to be inviscid and incompressible, while the motion was irrotational.

The transient response of liquid storage tanks due to external excitation can be strongly influenced by the interaction between the flexible containment structure and the contained liquid. Due to this fact, the amplitude of the sloshing depends on the nature, amplitude and frequency of the tank motion, liquid-fill depth, liquid properties, and tank geometry [21].

There are two major problems that arise in a computational approach to sloshing: the moving boundary conditions at the liquid–tank interface and excessive distortion of liquid elements, which may cause numerical instability or even computational incompleteness [22]. To include nonlinearity and avoid complex boundary conditions of moving walls, a moving coordinate system known as the ALE (Arbitrary Lagrangian Eulerian) method has been used [23]. The ALE method allows arbitrary motion of the moving boundary of the interaction of a rubberlike membrane and liquid with respect to their frame of reference by taking the convection of these points as described in [24–27]. Especially for 2D analysis where the remeshing of the liquid domain can be easily treated with the ALE description, numerical results were in good agreement with experimental results, see [28]. Currently, the mesh distortion and mesh adaptation problem can be resolved, to a large extent, by employing the ALE method with the help of a suitable remeshing and smoothing algorithm [29]. Referring to the papers by Soulaimani and Saad [30] and Cho and Lee [31], the boundary tracking in both Lagrangian and ALE approaches was straightforward because the liquid mesh moved exactly with the liquid particles.

The aim of this study was to clarify the nonlinear oscillation of the sloshing-like behavior such as a container made of a flexible membrane like a bladder tank. The two-dimensional interaction of a rubber-like membrane and liquid problem in a rectangular tank under horizontal vibration was examined by the arbitrary Lagrangian–Eulerian finite element method (ALE-FEM) and compared with the sloshing model to validate this theory. Furthermore, the influence of a large excitation and membrane tension were examined. The effects of finite deformation of the membrane, i.e., material and geometrical nonlinearities of the membrane and liquid were considered.

The outline of this paper after the introduction is as follows. In Section 2, the analytical model and conditional assumptions were introduced. Section 3 presented the constitutive equations of a rubberlike membrane that can be defined by a strain energy function as a hyperelastic material [32]. After that, equations were expressed in both the structural and liquid domains and were based on a closed variation setting. Section 4, the arbitrary Lagrangian–Eulerian finite element method (ALE-FEM) based on the Nitikitpaiboon–Bathe method [26] was explained and applied to the equations in Section 3. Then, mesh updating techniques were required for the ALE method because the liquid undergoes large deformations [33]. Mesh updating techniques for the liquid domain were defined for every analysis model at each time step. Lastly, the weak form of the dominant equation in Section 4 was discretized using a finite element method. Moreover, the Newmark-β method was used to address the time differential