Aerospace Science and Technology 50 (2016) 44–54

Contents lists available at ScienceDirect

Aerospace Science and Technology

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Effects of combined hardening and free-play nonlinearities on the response of a typical aeroelastic section

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ARTICLE INFO

Article history:
Received 2 April 2015
Received in revised form 18 December 2015
Accepted 19 December 2015
Available online 29 December 2015

Keywords:
Aeroelasticity
Hopf bifurcation
Flutter
Hardening nonlinearity
Free-play nonlinearity
Higher-order spectra

ABSTRACT

This paper presents an investigation about the dynamic response of a three-degree of freedom airfoil with hardening nonlinearity in the pitching stiffness and free-play nonlinearity in the control surface stiffness using bifurcation and HOS analysis. An experimental apparatus was conceived to test an airfoil aeroelastic responses when nonlinearities of varying intensities are present. A numerical model is also used to simulate at the same conditions of the experimental tests. It is based on the classical theory for the linear unsteady aerodynamics with corrections for arbitrary motions coupled to a three-degree of freedom typical aeroelastic section, where the hardening effect is modeled by means of rational polynomial function, while the free-play is represented by hyperbolic functions combination. Aeroelastic responses are analyzed from numerical and experimental results. Hopf bifurcations are identified and diagrams of amplitudes versus airspeeds are used to investigate the conditions in which the system is supercritical or subcritical. Higher-order spectra analysis is also used to check on frequency couplings, thereby allowing to identify quadratic- and cubic-like nonlinear behavior. The study of the phenomena associated with the hardening, free-play and their intensity variation effects may be useful in the mitigation of undesired responses of aeroelastic systems.

1. Introduction

The aeroelasticity is a multidisciplinary field of engineering science that deals with the mutual interaction between structural dynamics and unsteady aerodynamic loading [1]. Aeroelastic systems may behave nonlinearly, therefore exhibiting phenomena such as bifurcations, limit cycle oscillations (LCO), and chaos [2,3]. The source of the nonlinearities can be in structural dynamics and/or the unsteady aerodynamic loading and may be difficult to predict. Nonlinear unsteady aerodynamic loading can be the result of separated flows (viscous effects) or shock excursion (compressibility effects). Structural nonlinearities can arise from geometrical of material related effect, and classified as concentrated or distributed. Concentrated structural nonlinear effects can be incorporated into numerical models through the elastic restoring forces or moments representations, being the most common form of nonlinear inclusion to aeroelastic models. Typical concentrated structural nonlinear representations can be given by polynomial fitting functions, nonlinear damping effects, free-play, and hysteresis.

The literature in this field is quite vast, demonstrating that nonlinear aeroelastic problems in the aviation are of increasing importance in aircraft design. For example, the limit cycle oscillations have caused persistent aeroelastic problems in aircraft, such as the F-16, where the existence of hardening nonlinearity in wings' pitching moment stiffness was observed [4,5]. O'Neil and Strganac [6] have developed an experimental test that provides direct measurements from the typical aeroelastic section with cubic nonlinearity in the pitch and plunge motion. They examined the sensitivity of the response to system parameters and provided important conclusions on the effect of smooth nonlinear effect in aeroelastic response of airfoils. Recently, Vasconcellos et al. [7]

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http://dx.doi.org/10.1016/j.ast.2015.12.022
1270-9638/

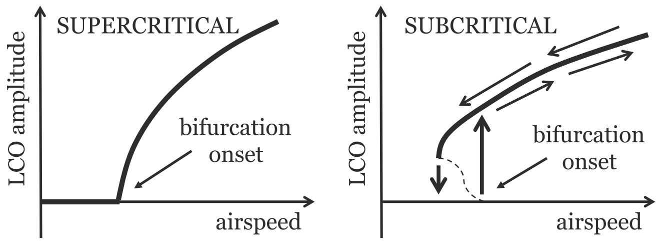

Fig. 1. Hopf bifurcation behavior.

have shown that the hyperbolic tangent function combination approach for modeling discontinuous nonlinearities is appropriate for detecting different nonlinear behaviors, including the experimentally observed LCO, chaos and transitions.

It is known that the system behavior is directly related to the nonlinearities involved, for example, system under free-play shows subcritical behavior [8–11]. The Hopf bifurcation appears when a stable system becomes unstable at certain parameter variation and the attractor becomes a LCO. There are two types of Hopf bifurcation: the supercritical Hopf bifurcation, where stable LCO appears after an unstable critical location, and subcritical Hopf bifurcation, where an unstable LCO is created among a stable critical location. Fig. 1 illustrates the possible behavior of the Hopf bifurcations. Supercritical bifurcations are sometimes called safe because the amplitude of the limit cycles grows up gradually as the parameter is increased after the bifurcation point [3]. In contrast, the subcritical bifurcations are called dangerous because in a subcritical system it is also possible that large-amplitude limit cycles suddenly appear as the parameter is varied. Typically subcritical behavior causes bistable behavior, i.e, there are different solutions when the flow velocity is increased and decreased near the bifurcation point.

Bifurcation analysis can be used to determine quantitative and qualitative changes in the system features, such as the number and type of solutions, under the variation of one or more parameters [12]. An example of this approach for a nonlinear aeroelastic system is given in Ref. [10] that analyzed changes in typical aeroelastic section behavior characterized by a cubic structural nonlinearity. It has been shown that the stability of the system is determined by the influence of the different nonlinear couplings.

In this paper, an investigation on the combined influence of hardening and free-play nonlinearities on the bifurcation response of a typical nonlinear section is presented. Numerical and experimental results are presented, thereby allowing comparisons and conclusions of the nonlinear features. Hardening nonlinearities with varying intensities in airfoil pitching motion and free-play (different gap values) in the control surface hinge were considered. Numerical model includes classical linear unsteady aerodynamics, while the equations of motion incorporates nonlinearities in the respective stiffness values. Traditional time integration method is used in aeroelastic simulations. The experimental apparatus was designed to allow two-dimensional typical section behavior in plunge, pitch and control surface motions, and the tests were performed using a open-circuit, blower-type low-speed wind tunnel. Bifurcation analysis comparing both numerical and experimental results was carried out. Higher-order spectral (HOS) analysis was performed to investigate the quadratic and cubic forms of nonlinear couplings due to the combined hardening and free-play effects.

The paper content includes a description of the mathematical model and the structural nonlinear representations, followed by the theoretical aspects of higher-order spectra analysis. A description of the experimental apparatus and test set-up is presented, where the main parameters used for numerical simulations are also presented. Then, bifurcation analysis results and higher-order spectra investigation for couplings are discussed and concluding remarks are presented.

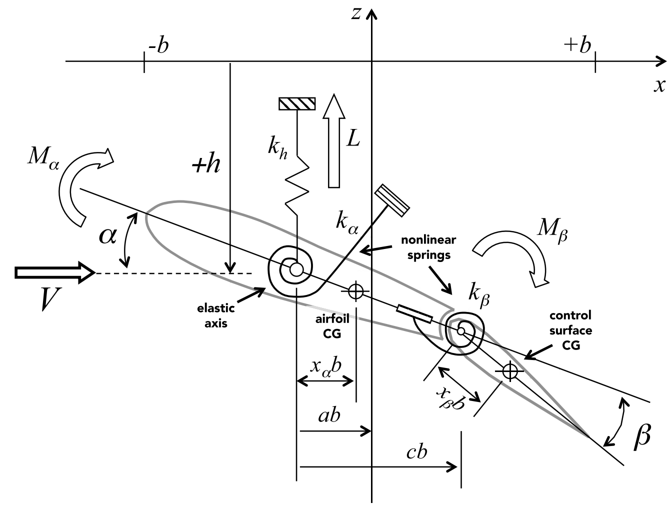

Fig. 2. Typical aeroelastic section representation.

2. Mathematical model

2.1. Aeroelastic equations

The mathematical model for typical section with three-degree of freedom (*cf.* Fig. 2) is derived admitting the basic principles given in Refs. [1,13], and the formulation is detailed in Ref. [7]. The resulting set of aeroelastic equations is,

$$\begin{bmatrix} (\frac{m_T}{m_W}) & x_\alpha & x_\beta \\ x_\alpha & {r_\alpha}^2 & [{r_\beta}^2 + (c-a)x_\beta] \\ x_\beta & [{r_\beta}^2 + (c-a)x_\beta] & {r_\beta}^2 \end{bmatrix} \begin{Bmatrix} \ddot{\xi}(t) \\ \ddot{\alpha}(t) \\ \ddot{\beta}(t) \end{Bmatrix} + \begin{bmatrix} d_{1,1} & d_{1,2} & d_{1,3} \\ d_{2,1} & d_{2,2} & d_{2,3} \\ d_{3,1} & d_{3,2} & d_{3,3} \end{bmatrix} \begin{Bmatrix} \dot{\xi}(t) \\ \dot{\alpha}(t) \\ \dot{\beta}(t) \end{Bmatrix} + \begin{bmatrix} {\omega_h}^2 & 0 & 0 \\ 0 & {r_\alpha}^2 {\omega_\alpha}^2 \frac{F(\alpha)}{\alpha(t)} & 0 \\ 0 & 0 & {r_\beta}^2 {\omega_\beta}^2 \frac{F(\beta)}{\beta(t)} \end{bmatrix} \begin{Bmatrix} \xi(t) \\ \alpha(t) \\ \beta(t) \end{Bmatrix} = \left(\frac{1}{b^2 m_W} \right) \begin{Bmatrix} -bL(t) \\ M_\alpha(t) \\ M_\beta(t) \end{Bmatrix}, \quad (1)$$

where b is the semi-chord of the airfoil, U is the airspeed, $h(t)$, $\alpha(t)$, and $\beta(t)$ are the plunge (positive downwards), pitch and control surface (trailing edge movable tab) displacements, respectively, $\xi(t) = \frac{h(t)}{b}$ is the non-dimensional plunge displacement, a is the distance in x-direction of the elastic axis position from the reference system origin in proportion of the airfoil semi-chord, c is the distance in x-direction of the control surface hinge position from the reference system origin also proportional to the airfoil semi-chord, x_α and x_β are the dimensionless distances from elastic axis respectively to the airfoil and the control surface centers of gravity (CG), r_α and r_β are the airfoil (with respect to the elastic axis) and control surface (with respect to the hinge line) radius of gyration, respectively, k_h, k_α, and k_β are the plunge, pitch, and control surface stiffness values, respectively, m_W is the wing (airfoil) weight, m_T is the total weight of the aeroelastic device considering all moving masses not immersed in the flowfield, $d_{i,j}$ are added structural damping factors with respect to each airfoil motion (Rayleigh approach), $L(t)$ is the unsteady lift force, $M_\alpha(t)$ and $M_\beta(t)$ are the unsteady pitch and hinge aerodynamic moments, respectively, $F(\alpha)$ and $F(\beta)$ are functions representing the nonlinearities related to pitch and control surface motions, respectively.