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Transitions in the unsteady wakes and aerodynamic characteristics of the flow past three square cylinders aligned inline

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ABSTRACT

A numerical study is performed to analyze the effect of low Reynolds numbers (*Re*) and gap spacings (g) on the flow around three square cylinders aligned inline using the multi-relaxation-time lattice Boltzmann method (MRT LBM). The Reynolds number is varied from $Re = 90$ to 175 while the spacing between the cylinders is taken in the range $0.5 \leq g \leq 6$. It is found that MRT LBM captures the flow characteristics efficiently. Seven different flow patterns are revealed in this study at different *Re* and g values. Variation of different aerodynamic force coefficients, like mean drag coefficient, Strouhal number, root mean square values of drag and lift coefficients, with flow patterns are discussed in detail in order to develop a link between flow patterns and force characteristics. The drag inversion (DI) spacing (or critical spacing) for the middle cylinder lies within the spacing range $2 \leq g \leq 3$ and depends on *Re*. The effect of *Re* on DI is also studied and it is concluded that the DI spacing decreases with increment in *Re*. At all chosen *Re* it is found that the Strouhal number decreases up to DI spacing value while it has increasing behavior after that. The negative values of average drag coefficient for third cylinder are also observed either at low spacing or at high spacing values. $Re = 90$ and 140 are found to be critical for different wake characteristics. Generally, it is observed that the upstream cylinder has higher drag force compared to the middle and downstream one but at some *Re* and g values the middle or downstream cylinder also found to have higher drag force than the upstream one. The root mean square value of lift coefficients for all the cylinders is higher than the corresponding root mean square value of drag coefficients.

1. Introduction

Flow around bluff bodies has attracted the attention of many researchers due to its practical engineering applications. Most of the work on bluff bodies is related to the study of flow induced vibrations, their resulting effects on the structures, dependence of wake patterns and aerodynamic force characteristics on different parameters like $Re = U_\infty d/\nu$, spacing $g = s/d$ (where s is the distance and d is size of cylinder) between the structures, as well as size and shape of structures. Various applications of bluff bodies can be found in cooling towers, cable suspension bridges, high-rise buildings, heat exchangers etc at high *Re* values while at low *Re* these applications can be found in the computer equipments, cooling of electronic devices and micro-electro-mechanical-systems (MEMS) etc. According to Vikram et al. [30] an initially smooth and steady flow across a square cylinder can bring damaging effects in case where there is a similarity between the natural frequency of the cylinder and the shedding frequency of vortices.

From literature survey it can be concluded that most of the researchers worked on the flow past single or two tandem cylinders experimentally as well as numerically. For single cylinder some of the studies are those of Dutta et al. [8], Rajani et al. [25], Mahir [21], and Saha et al. [27]. Dutta et al. [8] experimentally measured the flow patterns and aerodynamic forces in the wake of a square cylinder at different aspect ratios (AR) and orientations (θ) at $Re = 410$. They observed that the Strouhal number ($St = f_s d/U_\infty$) increases while the drag coefficient ($C_d = 2F_d/\rho U_\infty^2 d$) decreases with the increment in AR. The authors also observed a minimum value of drag and maximum *St* at $\theta = 22.5°$. Rajani et al. [25] numerically simulated the two- and three-dimensional flow over a circular cylinder for $Re = 0.1$ to 400 which covered the three different flow regimes i.e. creeping flow, steady closed near wake and the laminar vortex shedding. It was found that up to $Re = 200$ the two-dimensional ($2D$) computation results can be captured accurately while the three-dimensional ($3D$) effects appear beyond this value of *Re*. Mahir [21] performed $2D$ and $3D$ numerical simulations of a square cylinder placed near a wall and reported that the

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Nomenclature

AR	aspect ratio
C_d	drag coefficient
C_l	lift coefficient
c_1	upstream cylinder
c_2	middle cylinder
c_3	downstream cylinder
$Cdmean$	mean drag coefficient
$Cdmean1$	mean drag coefficient of upstream cylinder
$Cdmean2$	mean drag coefficient of middle cylinder
$Cdmean3$	mean drag coefficient of downstream cylinder
$Cdrms$	root-mean-square value of drag coefficient
$Cdrms1$	root-mean-square value of drag coefficient of upstream cylinder
$Cdrms2$	root-mean-square value of drag coefficient of middle cylinder
$Cdrms3$	root-mean-square value of drag coefficient of downstream cylinder
$Clrms$	root-mean-square value of lift coefficient
$Clrms1$	root-mean-square value of lift coefficient of upstream cylinder
$Clrms2$	root-mean-square value of lift coefficient of middle cylinder
$Clrms3$	root-mean-square value of lift coefficient of downstream cylinder
d	diameter of the cylinder
F_d	in-line force component
F_l	transverse force component
f_s	vortex shedding frequency
g	gap spacing
H	height of the computational domain
L	length of the computational domain
Ld	downstream position
Lu	upstream position
Re	Reynolds number
s	surface-to-surface distance between cylinders
St	Strouhal number
$St1$	Strouhal number of upstream cylinder
$St2$	Strouhal number of middle cylinder
$St3$	Strouhal number of downstream cylinder
$U\infty$	uniform inflow velocity

Greek symbols

ν	kinematic viscosity
ρ	fluid density

3D simulations predict lower mean drag coefficients and square root of lift coefficients compared to those observed in 2D simulations at $Re = 175$ and 185. Saha et al. [27] numerically examined the transitions in the wake of a square cylinder. They concluded that in the range $Re = 40$ to 600, the flow undergoes from the steady state (at $Re = 40$) to the chaotic one at $Re = 600$. Between these two states the periodic, quasi-periodic, frequency locking phenomen hold. They also found that each transition has a critical Re. Further work on single cylinder can be found in [1,3,15,17,23,24].

Among others the work of Zdravkovich [32], Wang et al. [31], and Deng et al. [5] can also be cited for two tandem (inline) cylinders. Zdravkovich [32] classified the flow patterns for tandem arrangement in to the single slender body, alternate reattachment, quasi-steady reattachment, intermittent shedding, and binary vortex shedding regimes depending on Re and g values. Wang et al. [31] experimentally measured the flow around two tandem circular cylinders in a horizontal soap film tunnel at $Re = 60, 80, 100$ and $g = 1$ to 12. They observed different wake patterns such as single bluff body, shear layer reattachment, synchronization of vortex shedding and secondary vortex formation at different Re and g values. The authors also reported that the averaged drag of all the cylinders was less than single cylinder values. Deng et al. [5] reported that at $Re = 220$ the 3D effects appear in the flow for $g \geq 4$ while for $g \leq 3.5$ flow keeps 2D effects while studying 3D flow around two tandem circular cylinders. It was also observed that the spacing range $3.5 < g < 4$ is critical for appearance of 3D instability. It was found in previous studies around tandem bodies that the upstream cylinder experiences higher lift than the downstream one and the vortex shedding frequency decreases by introducing second cylinder either in the upstream or downstream position of first cylinder [30]. More work on two tandem bodies can be found in [7,9,14,20,26] and references therein.

With introduction of more than one body in the wake of a single body the phenomenon of reattachment of shear layers may be completely different from those observed for single body placement. Because more the number of bodies (cylinders) in flow field longer will be the shear layer resulting drastic changes in the aerodynamic forces. It can be explored from the literature survey that much less attention is given to the work on flow past more than two bodies in inline arrangement. Some of the studies are those of Igarashi and Suzuki [13], Harichandan and Roy [11], Vasel-Be-Hagh et al. [29], Bao et al. [2], and Sewatkar et al. [28]. Igarashi and Suzuki [13] experimentally investigated the flow around three circular cylinders arranged in-line in the range $1.09 \times 10^4 \leq Re \leq 3.92 \times 10^4$ and classified the wake patterns according to the behavior of shear layers emerging from the first cylinder. They found that the most upstream cylinder experiences high drag force compared to the downstream ones while the middle cylinder has lowest drag compared to other two cylinders. Moreover they found that the Re value at which abrupt changes in flow characteristics occurs is in inverse relation with spacing. Their relation is given by $Re_c = 5.2 \times 10^4 \times (g)^{-3.8}$ and they named it 'reattachment Reynolds number'. Harichandan and Roy [11] performed numerical investigations on the flow past two and three circular cylinders arranged side-by-side and inline at $Re = 100$ and 200. For inline arrangement they observed that at $g = 2$ a steady wake pattern occurs at $Re = 100$ while sparse Karman street was observed at $Re = 200$ at same value of g. With increment in the gap value to $g = 5$ no separation or reattachment of shear layers from upstream cylinder to downstream one was seen instead a Karman vortex streets developed between the cylinders. Also the authors observed distinct identical shedding frequencies for all the cylinders. Vasel-Be-Hagh et al. [29] reported that $Re = 42$, 63 and 150 are critical Re for pressure, viscous and total drag forces respectively at $g = 2$ in his work on a tandem unit of three circular cylinders. Moreover it was also found that the flow mode transition from steady to unsteady state starts between $Re = 61$ to 105. Also the vortex shedding frequency and Strouhal number increase with Re. Bao et al. [2] worked on an inline array of square cylinders and stated that the flow mode changes from steady to unsteady state when the spacing crosses a critical value. Sewatkar et al. [28] examined the flow around six inline square cylinders both experimentally as well as numerically at $0.5 \leq g \leq 10$ and $80 \leq Re \leq 320$. They found that at $Re = 100$ flow can be classified to different regimes depending on spacing. These regimes were synchronous ($0.5 \leq g \leq 1.1$), quasi-periodic-I ($1.2 \leq g \leq 1.3$), quasi-periodic-II ($1.4 \leq g \leq 5$) and chaotic ($6 \leq g \leq 10$). They also observed that