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Convective heat transfer and thermo-diffusion effects on flow of nanofluid towards a permeable stretching sheet saturated by a porous medium

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ABSTRACT

Boundary layer flow of a nanofluid towards a permeable stretching surface saturated by porous medium with a convective boundary condition is considered. After employing viable similarity transforms, equations governing the flow are reduced to a system of nonlinear ordinary differential equations. Well known numerical technique Runge–Kutta–Fehlberg method is applied to solve the system. Concrete graphical analysis is carried out to study the effects of different emerging parameters on velocity, temperature, solute and nanoparticle concentration profiles coupled with a comprehensive discussion. Tables are also constructed to compare current study with already existing results in the literature. Numerical effects of local Nusselt number, local Sherwood number and nanofluid Sherwood number are also discussed.

1. Introduction

The fluids like water, kerosene oil, ethylene and glycol are commonly used in many heating and cooling systems. These fluids (called the base fluids) mostly are very poor conductors of heat. To enhance the performance of such systems we need to handle the problems caused by their poor conductivities. To boost the thermal conductivity and other thermal properties, a relatively new technique is being applied nowadays. They add an appropriate amount of nano-sized particles of good conductors to the base fluids which enables them to bear relatively higher thermal conductivities. Choi [1] was the first one to come up with the idea that by adding nano-sized particles of good conductors such as copper, aluminum, titanium, iron and other oxides to the fluids, thermal properties of nanofluids can be enhanced. Choi [2] further established that the nanoparticles can be added to conventional fluids to increase the thermal conductivities and other thermal properties of these fluids. These enhancements can practically be used in electronic cooling, heat exchangers, double plane windows, etc. A more comprehensive model was presented by Buongiorno [3] that incorporates all the nanotechnology based fluids that unveils the thermal properties superior to base fluids. He discussed all the convective properties of nanofluids by developing a more generalized model.

Boundary layer flows, due to their real world applications, have got much attention by the researchers in last few decades. These applications include, glass fiber production, engineering melt spinning, manufacturing of rubber sheets, etc. Khan and Pop [4] were the first to extend the general boundary layer flow problem presented by Crane [5] over a stretching sheet to the case of nanofluids. They used the Buongiorno's model to study the thermophoresis, Brownian motion and other thermo-physical properties of nanofluids. That study revealed a new insight of boundary layer flows. Since then, more researchers are now interested in studying the flow behavior in different geometries incorporating nanofluids. Nield and Kuznetsov [6,7] used the same model to investigate several problems related to nanofluids. Makinde and Aziz [8] generalized the Khan and Pop's problem to convective boundary condition instead of isothermal condition. Following Makinde and Aziz, several authors have used convective boundary condition to revisit the problems which were previously studied under isothermal or isoflux boundary conditions. They also extended these studies to the case of nanofluids. One can easily find enough literature on convective nanofluid flows, however some of the studies are reported in [9–24] and references therein. Those also include some studies on nanofluid flow over a permeable stretching sheet saturated by porous medium.

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Nomenclature

k_1	Permeability of porous medium
c_f	Skin friction coefficient
C_w	Solutal concentration at the surface
T_w	Temperature at the surface
$\hat{\phi}_w$	Nanoparticle volume fraction at the surface
C_∞	Ambient solutal concentration
T_∞	Ambient temperature
$\hat{\phi}_\infty$	Ambient nanoparticle volume fraction
U_∞	Free stream velocity
u, v	Velocity components along x and y directions
f	Dimensionless stream function
s	Dimensionless concentration
D_T	Thermophoresis diffusion coefficient
D_S	Solutal diffusivity
D_B	Brownian motion diffusion coefficient
D_{TC}	Dufour diffusivity
D_{CT}	Soret diffusivity
Pr	Prandtl number
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Nd	Modified Dufour parameter
Le	Lewis number
Ld	Dufour solutal Lewis number
Ln	Nano Lewis number
Nu_x	Local Nusselt number
Sh_x	Sherwood number
Shr_x	Nanofluid Sherwood number
Nur	Reduced Nusselt number
Shr	Reduced Sherwood number
$Shrn$	Reduced nanofluid Sherwood number

Greek symbols

η	Dimensionless similarity variable
μ	Dynamic viscosity of the fluid
υ	Kinematic viscosity of the fluid
$(\rho)_f$	Density of the base fluid
$(\rho c)_f$	Heat capacity of the base fluid
$(\rho c)_p$	Effective heat capacity of the nanoparticle
ψ	Stream function
α_m	Thermal diffusivity
σ	Electric conductivity
θ	Dimensionless temperature
ϕ	Dimensionless nanoparticle volume fraction
τ	Parameter defined by $\frac{(\rho c)_p}{(\rho c)_f}$

Subscripts

∞	Condition at free stream
w	Condition at the wall

Until now, all aforementioned studies were conducted in the absence of Soret and Dufour effects. In a flowing liquid, simultaneous occurrence of heat and mass transfer results in a complicated relationship between the fluxes and the flowing nature of the fluid. Energy flux can be generated not only by the temperature gradients but also by the composition gradients. Mass fluxes can be created by the temperature gradients that results in Soret (thermo-diffusion) effect. On the other hand, the energy fluxes caused by the composition gradients are termed as Dufour (diffusion-thermo) effect. When there is a density difference in the flow regime, such fluxes play a significant role. For the flows of mixture of gases with light molecular weights (He, H_2) and moderate weights (N_2, air), Soret and Dufour effects cannot be neglected. Awad et al. [21] used the same idea to study the thermo-diffusion effect on the flow of nanofluid over a stretching sheet that has later been extended for power law stretching sheet by Goyal et al. in a recent study [25]. Khan et al. [26] also used the same concept and studied the thermo-diffusion effects on stagnation point flow towards a stretching sheet. Nowadays, aerospace technology has not only enabled us to room around our own space but also to explore the outer space. During the exploration or traveling, crafts and gear go through structural loads, faces certain conditions like variation in dynamic pressure, temperature and electric/magnetic fields. As the craft manufacturing is a highly complex procedure, many branches of science are applied and used in the process. One the most used is fluid mechanics. Aerodynamics plays a vital role in any flight and understanding it correctly is very important. Air's flow through the wings and wind tunnels can be understood by utilizing the concepts of fluid mechanics. Lubrication, fuel supply and some cooling methods totally relay on deep understanding of fluids involved.

Heat convection, and conduction, also occurs in certain parts of aircraft. Extra heat is generated due to the friction, chemical processes or combustion of flues. On one hand, it may be useful and necessary to run the machines, but on the other, an imbalance in generated can lead to disaster. Thermo-diffusion and diffusion-thermo phenomena occurs quit often. So, we hope that our study shall cast some light on different aspects of these effects. By understanding the flow of nanofluids around the surface of the planes, scientists and engineers would be able to enhance the performance of the aircrafts furthermore.

Literature survey reveals that no study is available discussing thermo-diffusion effects on a permeable stretching sheet saturated by a porous medium with convective boundary condition. To fill out this gap, a concrete analysis is provided in current study. Main objective of this study is to extend the work of Awad et al. for a permeable sheet incorporating the porous medium. Convective boundary condition is considered instead of isothermal condition. Mathematical formulation of the problem is carried out to study the combined effects of thermophoresis, Brownian motion and thermo-diffusion. After employing viable similarity transformations, equations governing the flow are converted to system of ordinary differential equations. Shooting technique is used to convert boundary value problems to a system of initial value problems. A well-known numerical technique called the Runge–Kutta–Fehlberg method is then used to solve the resulting system. Influence of emerging parameters on velocity, temperature, solute and nanoparticle concentration profiles are presented with the help of graphs and tables coupled with a comprehensive discussions. Also, a comparison of current study to the previous ones is provided to authenticate our solutions.

2. Governing equations

Consider the two-dimensional, incompressible, steady and laminar flow of a nanofluid over a permeable sheet embedded in a uniform porous medium. Sheet is being stretched with a linear velocity varying with the distance x, i.e. $U_w = ax$, where a is a real positive number and x is the coordinate measured from the location where the sheet velocity is zero. The sheet's surface temperature T_w (to be determined later), is a result of convective heating process characterized by the fluid temperature T_f and heat transfer coefficient h_f. C_w is the solutal concentration and ϕ_w is the nanoparticle concentration at the wall. At a large distance from the sheet; temperature, solutal concentration and the