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A layerwise semi-analytical method for modeling guided wave propagation in laminated and sandwich composite strips with induced surface excitation

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ABSTRACT

A semi-analytical method for the quick and robust solution of guided waves excited in infinite laminated composite strips is presented. In the proposed method, the response of the strip is constructed in time-space domain directly from the governing wave equations, assuming a layerwise variation of the through-the-thickness displacement components. Analytical Fourier transforms, with respect to time and space, provide the transformed response in the frequency-wavenumber domain. Dispersive modal properties for the structure are extracted. Finally, the time transient response of the propagative guided waves is calculated using two inverse Fourier transforms, the first is calculated via the Cauchy's residue theorem, whereas the second is numerically computed. Various numerical results are presented for isotropic, cross-ply composite laminate and sandwich strips and are compared with results from other reported methods as well as with experimental results in composite strips.

1. Introduction

The extensive application of composite materials in aerospace, civil engineering, transport and renewable energy structures requires new structural health monitoring (SHM) techniques capable of revealing and locating damage in composite structures and ensuring their structural sustainment. Today, the guided-wave based applications gain enormous interest in the industry for material characterization and design, non-destructive evaluation (NDE) and SHM. The generation and monitoring of guided Lamb waves generated using various excitation and sensing methods (e.g. NDE transmitters/receivers, permanently attached piezoelectric wafer or film actuators and sensors) seems to be one of the most encouraging methods for the detection, identification and localization of damage in composite structures [1]. Yet, the modeling and design of such systems is challenging, since all the available techniques involve high-frequency, high wavenumber excitation signals, which makes their numerical modeling computationally inefficient.

Early work in the area of guided wave propagation as a technique of damage detection was focused on the dispersive behavior of guided waves in laminated strips and plates of finite thickness, showing that the dispersive modal propagation behavior is strongly influenced by the anisotropic properties of each lamina and the stacking sequence. Various previous works [2–5] have studied the propagation of wave modes in multilayered structures.

Over the last few decades finite element methods (FEM), boundary element methods (BEM), and spectral element methods (SEM), have been developed for the simulation of elastic wave propagation in various media. Many of the previous numerical methods require high in-plane spatial and temporal discretization to model high-frequency, high-wavenumber ultrasonic waves and, at the same time, suffer from numerical shortcomings (high time-of-flight, artificial dispersion, spurious wave modes, and so forth). Finite Element Analysis was used by Mukdadi and Datta [6] in order to study the guided waves in finite-width elastic plates. Marzani [7] used an enhanced spectral finite element (SFE) developed for axially symmetric waveguides in which material damping is taken into account. Similar SFE based formulations have been used to calculate the time-transient response in anisotropic plates [8–10], functionally graded cylinders [11], rails [12], transversely isotropic composite plates [13] and helical waveguides [14]. Crack detection in isotropic structures via modeling high-frequency wave propagation was attempted by Coccia et al. [15,16] in arbitrary cross-sectional waveguides, where certain modes were selected for

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the detection of surface cracks. Giurgiutiu [17] modeled surface attached piezoelectric actuators and sensors to excite and detect tuned Lamb waves for structural health monitoring. Gresil and Giurgiutiu [18] developed a solution method to predict the one-dimensional guided-wave propagation in 2D strips which combines an exact analytical solution at some segments of the strip with a FEA numerical solution at other segments containing damage or other non-uniform characteristics. Ahmat, Vivar-Perez and Gabbert [19] developed a semi-analytical method which uses linear finite element interpolation to approximate the wave behavior in the plate thickness direction while in the wave propagation direction the displacement field is analytically described by a complex exponential. The previous semi-analytical Finite Element method was subsequently used for the description of leaky Lamb wave calculation, extracting dispersion curves [20]. An early work of Ait Yahia et al. [21] considers the wave propagation in plates with porosities and functionally graded materials, using higher-order shear deformation plate theories.

On the other hand, exact analytical solutions for the free-vibration and wave propagation in composite plates with piezoelectric actuators have been reported by Heyliger and Saravanos [22], Bottai and Giurgiutiu [23], Cesnik et al. [24] and Banerjee et al. [25] which resolve the previous computational and numerical issues and provide valuable benchmark results, but are cumbersome and time consuming to formulate. Sanderson and Catton [26] presented an analytical modeling technique for the simulation of long-range ultrasonic guided waves in prismatic structures. Other analytical methods utilize the global matrix method to investigate the dispersion characteristics of propagating guided wave modes in multilayered composite laminates due to transient surface excitations [27], as well as, to produce accurate displacement time histories for wave propagation in laminated plates [28]. Glushkov et al. [29] used a mathematical model based on an integral approach for the prediction of non-axisymmetric and frequency-dependent generated wavefields from piezoelectric disc actuators with wrapped electrodes. Tounsi et al. [30] and Bourada et al. [31] reported a refined trigonometric shear deformation theory taking into account transverse shear deformation effects for the thermoelastic bending analysis of functionally graded sandwich plates, and the bending and vibration of functionally graded beams, respectively.

Although pure analytical approaches offer fast and accurate results, their formulation for laminated composite plates is complicated and cumbersome, hence are effectively limited to rather simple geometries and material configurations. On the other hand, common numerical methods, such as Finite Element and Finite Difference methods, require very high spatial discretizations over the plane of the plate and through the thickness, to yield converging solutions in terms of wave dispersion data and group velocities. Particularly, at high ultrasonic frequencies where more complex wave modes are excited, the application of numerical methods becomes a computationally intractable task, involving extremely high number of elements.

The present paper offers an intermediate solution, and aims to further improve the robustness, accuracy and computational speed of semi-analytical methods for the prediction of guided waves in laminated composite plates, by taking advantage of various available layerwise laminate models for the representation of the through-thickness fields, and combining them with an analytical solution for the in-plane wave propagation. The proposed work focuses on the Structural Health Monitoring and damage detection in composite aerospace structures with guided waves methods. Generalized 3D layerwise laminate models [29–32,32–35] have been developed to overcome shortcomings of FE interpolation methods in the prediction of the complex through-thickness displacement and stress fields in thick composite laminates with and without piezoelectric layers, hence, are expected to be advantageous for the analysis of high-frequency/high-wave-number antisymmetric and symmetric guided waves. The novelty in the present paper is that it introduces 3D layerwise laminate plate theories in semi-analytical hybrid solutions of laminated composite plates and demonstrates their accuracy and robustness. Additionally, different benchmark cases are presented, which have not been studied yet.

In the remaining sections, the generalized equations of motion are first formulated. The layerwise theory presented by Robins and Reddy [33] for composite beams and latter extended by Saravanos and Heyliger [32] to laminated piezocomposite beams and plates is used. Surface traction excitations are considered in the equations to model time-varying external transverse forces and/or shear tractions of finite length simulating piezoelectric actuators. The system equations are transformed into the frequency-wavenumber domain, using a two-dimensional Fourier transformation, and the solution is obtained. The method of summation of the modal data weighted with the spectrum of applied load [6] is used for the calculation of the frequency response of the strip. The time domain response is subsequently recovered by the application of the inverse Fourier transform on the waveguide's frequency response. Obtained solutions for an unbounded infinite strip with point load excitation are compared with results reported in literature [7,25] in order to evaluate the performance of the method. Additional results with excitation via piezoelectric actuator are compared with experimental results. Finally, results are presented for laminated and sandwich composite strips to demonstrate the robustness of the method and to investigate the effect of lamination on the wave characteristics.

2. Theoretical formulation

2.1. Governing equations of the problem

In this paper we consider the propagation of straight-crested guided waves along a multilayered strip of arbitrary lamination, of infinite length and total thickness h (Fig. 1a). The strip consists of N_p physical layers or plies through the thickness, perfectly bonded together. The geometry of the strip is referred to a Cartesian coordinate system, in which the x axis coincides with the length and the z axis is parallel to the thickness of the strip.

The consideration of straight-crested waves implies plane strain conditions in the xz plane, i.e. $\varepsilon_y = \varepsilon_{xy} = \varepsilon_{yz} = 0$, and negligible variation of the non-vanishing strains and stresses in the y-direction. Hence, the linear stress-strain constitutive relations for a rotated composite ply are deducted from the three-dimensional constitutive equations (see Appendix) and are given by,

$$\boldsymbol{\sigma} = [\mathbf{C}]\boldsymbol{\varepsilon} \tag{1}$$

where, $\boldsymbol{\sigma} = \{\sigma_x, \sigma_z, \sigma_{xz}\}^T$ and $\boldsymbol{\varepsilon} = \{\varepsilon_x, \varepsilon_z, \varepsilon_{xz}\}^T$ are the extended stress and strain vectors; $\sigma_x, \sigma_z, \sigma_{xz}$ are the normal and shear stresses; and the ply stiffness matrix $[\mathbf{C}]$ is,

$$[\mathbf{C}] = \begin{bmatrix} C_{11} & C_{13} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{55} \end{bmatrix}$$

The stress equilibrium equations in the x and z direction provide the equations that describe the motion of the straight-crested wave. Considering that the derivatives of stresses with respect to y are negligible, the stress equilibrium equations take the form,

$$\begin{aligned} &\sigma_{x,x}(x,z,t) + \sigma_{xz,z}(x,z,t) = \rho\ddot{u}(x,z,t) \\ &\sigma_{xz,x}(x,z,t) + \sigma_{z,z}(x,z,t) = \rho\ddot{w}(x,z,t) \end{aligned} \tag{2}$$