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Coplanar ground-track adjustment using time difference

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A B S T R A C T

The coplanar ground track adjustment problem is studied for both impulsive and continuous low-thrust maneuvers considering the J_2 perturbation. For the initial orbit, semi-analytical solutions of the flight time and the longitude difference are obtained for an assigned ground-site latitude. Based on the time difference, the flight time for the two-body model is derived. This avoids directly solving the transfer orbit with the J_2 effect. Then two methods including single-impulse method for free final orbit and shape-based low-thrust method for an assigned final orbit are proposed. For the single-impulse method, the analytical approximate solution is obtained by solving a cubic equation. For the low-thrust method, the shape-based approach is used with a combined inverse polynomial and trigonometric function. The results of several numerical examples show that the final longitude errors of the proposed methods are less than 0.04 deg for the nonlinear J_2 model.

1. Introduction

Responsive space aims to provide more flexible and more affordable space applications in a timely manner to users. Thus, time and cost are two main considerations in responsive space. There are usually two distinct ways to improve space responsiveness: the first one is to quickly develop and launch a new satellite, whereas the other is to adjust the ground track of on-orbit satellite to cover the user-specified site. When a specified military or civil ground site (e.g., where natural disaster occurs) needs to be rapidly visited, it is better to employ the ground-track-adjustment strategy which saves more cost and can be realized in few days. Since the out-plane maneuver costs more fuel, only the coplanar maneuver is considered in this paper.

There are two maneuver ways including impulse maneuver (e.g., chemical propulsion) and low-thrust maneuver (e.g., electric propulsion) to fulfill the ground-track-adjustment mission according to the engine types. For the impulse maneuver, Zhu et al. [1] gave a hybrid Particle Swarm Optimization and Differential Evolution algorithm to solve the impulse ground track adjustment problem for multiple satellites. Moreover, Co et al. [2] presented a method to quantify the terrestrial distance using chemical or electric propulsion for given ΔV and available maneuvering time. It could be used to calculate the required impulse and thrust magnitude for a given distance. In addition, Co and Black [3] proposed a constant along-track low-thrust method for ground track adjustment. The constant acceleration magnitude was obtained by solving the time difference equation between the maneuvering and reference ground tracks. However, in these studies, the required impulse or thrust magnitude was not obtained in explicit form.

In the proceeding methods, there is no constraint on the final orbit whose ground track passes through the site. If the final orbit is a one-day repeated ground track one, it is particularly useful to observe the specified site every day [4]. The method in [3] can be extended to special final altitude orbit (e.g., repeated ground track) considering the thrust-coast period, but it is only valid for circular orbits. For final elliptic orbits, if the satellite point over the specified site is at the perigee, then higher ground resolution will be obtained for the same camera.

For the continuous low thrust case for an assigned final orbit, the shape-based method is a widely used approach. In the shape-based method, the low-thrust trajectory is supposed to be a specified-form shape function, in which the coefficients are obtained from boundary conditions. This approximation result is near to the "true" optimal low-thrust trajectory by numerical optimization algorithms. Many shape functions were proposed including exponential sinusoid [5], inverse polynomial [6], Fourier series [7], and polynomial semimajor axes [8]. Moreover, a three-dimensional trajectory shaping method was proposed in spherical coordinates [9]. However, these shape-based methods are only valid for the two-body problem such that it cannot be directly used for the ground track adjustment considering the J_2 perturbation.

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This paper studies the coplanar orbit adjustment problem for both circular and elliptic orbits considering the J_2 perturbation. The transfer time for the two-body model to the satellite point of the ground site is obtained using the time difference. Then a single-impulse method for free final orbit and a shape-based low-thrust method for an assigned final orbit are proposed to accomplish the mission.

2. Ground track calculation

When the initial orbital elements are given, the ground track (i.e., the latitude and the longitude) is determined using orbit propagation. Once the orbital elements at each time instant are obtained, the position vector in the Earth Centered Inertial (ECI) frame is

$$\mathbf{r}_{\text{ECI}} = \frac{a(1-e^2)}{1+e\cos f}\begin{bmatrix} \cos\Omega\cos(\omega+f) - \sin\Omega\sin(\omega+f)\cos i \\ \sin\Omega\cos(\omega+f) + \cos\Omega\sin(\omega+f)\cos i \\ \sin(\omega+f)\sin i \end{bmatrix} \tag{1}$$

where a is the semimajor axis, e is the eccentricity, i is the inclination, ω is the argument of perigee, Ω is the right ascension of ascending node, and f is the true anomaly. Then transforming it into the Earth-Centered Earth-Fixed (ECEF) frame yields

$$\mathbf{r}_{\text{ECEF}} = R_Z(\alpha_{Gt})\mathbf{r}_{\text{ECI}} \tag{2}$$

where R_Z is the rotating matrix around the coordinate axis Z in the ECI frame, and α_{Gt} denotes the Greenwich mean sidereal time (GMST) at the current time. The value of α_{Gt} is obtained by

$$\alpha_{Gt} = \alpha_{G0} + \omega_{\oplus} t \tag{3}$$

where $\omega_{\oplus} = 7.2921158553e{-5}$ rad/s, and the initial GMST is

$$\alpha_{G0} = 67310.54841^s + (876600^h + 8640184.812866^s)T_{UT1} + 0.093104T_{UT1}^2 - 6.2\times 10^{-6}T_{UT1}^3 \tag{4}$$

where T_{UT1} is the number of Julian centuries elapsed from the epoch J2000 and is computed using the Julian day numbers

$$T_{UT1} = \frac{JD_{UT1} - 2451545.0}{36525} \tag{5}$$

The Julian date is computed by [10]

$$JD_{UT1} = 367year - \text{INT}\left\{\frac{7[year + \text{INT}(\frac{month+9}{12})]}{4}\right\} + \text{INT}\left(\frac{275month}{9}\right) + day + 1721013.5 + \frac{\frac{second/60+min}{60} + hour}{24} \tag{6}$$

where the year, month, day, hour, minute, and second are Gregorian universal coordinated time.

Using the position vector $\mathbf{r}_{\text{ECEF}}$ in the ECEF frame, the latitude is computed by

$$\sin\varphi = \frac{\mathbf{r}_{\text{ECEF}}(3)}{r} = \sin(\omega+f)\sin i \tag{7}$$

and the longitude is

$$\lambda = \text{atan2}(\mathbf{r}_{\text{ECEF}}(2), \mathbf{r}_{\text{ECEF}}(1)) \tag{8}$$

3. Fight time and longitude difference for assigned latitude

Assume that a responsive satellite is moving on a known initial orbit, single impulse or continuous low-thrust maneuver is required to change its ground track to cover the user-specified ground site, S, whose longitude and latitude are λ and φ, respectively. For the nonmaneuvering initial orbit, this section provides a semi-analytical method to solve the flight time and the longitude difference for the latitude φ considering the linear J_2 effect.

For the assigned latitude φ of the target site, the argument of latitude at the corresponding sub-satellite point is obtained from Eq. (7) as

$$u_t = \omega + f_t = \sin^{-1}\left(\frac{\sin\varphi}{\sin i}\right) \tag{9}$$

The two values of Eq. (9) are associated to the ascending and descending orbit parts, respectively.

In this paper, the mean semimajor axis is denoted by $\bar{a}$, and the orbital elements without bar overhead indicate osculating elements. For the initial orbit with the J_2 perturbation, the linear transformation from osculating to mean semimajor axis is [11]

$$\bar{a} = a\left\{1 - \frac{J_2 R_{\oplus}^2}{2a^2}\left[(3\cos^2 i - 1)\left(\frac{a^3}{r^3} - \frac{1}{(1-e^2)^{3/2}}\right) + 3(1-\cos^2 i)\left(\frac{a}{r}\right)^3\cos(2\omega + 2f)\right]\right\} \tag{10}$$

where $R_{\oplus} = 6378.14$ km is the Earth radius and $J_2 = 1.082627e{-3}$. Ignoring the periodic term of the J_2 perturbation, the orbital elements a, e and i remain constant, the mean change rates of other elements are

$$\begin{cases} \dot{\omega}_{J_2} = C_{J_2}\left(2 - \frac{5}{2}\sin^2 i\right)/(1-e^2)^2 \\ \dot{\Omega}_{J_2} = -C_{J_2}\cos i/(1-e^2)^2 \\ \dot{M}_{J_2} = C_{J_2}\left(1 - \frac{3}{2}\sin^2 i\right)/(1-e^2)^{3/2} \end{cases} \tag{11}$$

where $C_{J_2} = 1.5J_2R_{\oplus}^2\sqrt{\mu}\bar{a}^{-7/2}$ and μ is the standard gravitational parameter. Then the other elements after time t become

$$\begin{cases} \omega_t = \omega + \dot{\omega}_{J_2}\cdot t \\ \Omega_t = \Omega + \dot{\Omega}_{J_2}\cdot t \\ M_t = M_0 + (\sqrt{\mu/\bar{a}^3} + \dot{M}_{J_2})\cdot t \end{cases} \tag{12}$$

If the initial orbit is circular, the argument of latitude with the linear J_2 effect is

$$u_t = u_0 + t\sqrt{\frac{\mu}{\bar{a}^3}} - 2\pi N_R + (\dot{M}_{J_2} + \dot{\omega}_{J_2})t \tag{13}$$

where N_R is the revolution number and u_0 is the argument of latitude at the initial time. Then the flight time with the linear J_2 effect is

$$t_{J_2} = \frac{(u_t - u_0) + 2\pi N_R}{\dot{M}_{J_2} + \dot{\omega}_{J_2} + \sqrt{\mu/\bar{a}^3}} \tag{14}$$

where the final orbit argument u_t is obtained by Eq. (9).

If the initial orbit is elliptic, the flight time with the J_2 effect is obtained from

$$M_t - M_0 + 2\pi N_R = \left(\sqrt{\frac{\mu}{\bar{a}^3}} + \dot{M}_{J_2}\right)t_{J_2} \tag{15}$$

Mean anomaly can be written as a function of eccentric anomaly, which is obtained from true anomaly

$$M = E - e\sin E \tag{16}$$