Aerospace Science and Technology 48 (2016) 186–192

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Nonlinear radiation effects on squeezing flow of a Casson fluid between parallel disks

Syed Tauseef Mohyud-Din *, Sheikh Irfanullah Khan

Department of Mathematics, Faculty of Sciences, HITEC University, TaxilaCantt, Pakistan

A R T I C L E   I N F O

Article history:
Received 6 June 2015
Received in revised form 10 October 2015
Accepted 18 October 2015
Available online 30 October 2015

Keywords:
Homotopy analysis method (HAM)
Casson fluid
Nonlinear radiation effect
Squeezing flow
Parallel disks

A B S T R A C T

In this paper, we investigate the nonlinear radiation effects in a time-dependent two-dimensional flow of a Casson fluid squeezed between two parallel disks when the upper disk is taken to be impermeable and the lower one is porous. Suitable similarity transforms are employed to convert governing partial differential equations into the system of ordinary differential equations. A well known Homotopy Analysis Method (HAM) is employed to obtain the expressions for velocity and temperature profiles. Effects of different physical parameters such as squeeze number S, Eckert number Ec and the dimensionless length on the flow when keeping $Pr = 7$ are also discussed with the help of graphs for velocity and temperature coupled with comprehensive discussions. Mathematica Package BVPh2.0 is utilized to formulate the total error of the system for both the suction and injection cases. The skin friction coefficient and local Nusselt number are presented with graphical aids for emerging parameters.

© 2015 Elsevier Masson SAS. All rights reserved.

1. Introduction

Squeezing flow between parallel disks is an important area of interest because of its application in biofluid mechanics like pumping of heart, flow through certain arteries, polymer industry process, injection modeling, compression, liquid-metal lubrication, and the squeezed films in power transmission. Most frequently, squeezing flows were illustrated in modeling of metal and plastic sheets, thin fiber and paper sheets formations etc. After the pioneer work done by Stefan (1874) [1], several attempts are reported that extended the traditional problem to heat transfer case. Different studies are available in literature that used various solution schemes to get analytical and numerical solutions for the said problem. Siddiqui et al. [2] examined a two-dimensional MHD squeezing flow between parallel plates. For parallel disk a similar problem has been discussed by Domairry and Aziz [3]. Both used the Homotopy perturbation method (HPM) to determine the solution.

Joneidi et al. [4] studied the mass transfer effect on squeezing flow between parallel disks using Homotopy analysis method (HAM). Most recently the influence of heat transfer in the MHD squeezing flow between parallel disks has been investigated by T. Hayat et al. [5]. They used HAM to solve the resulting nonlinear system of ordinary differential equations. Since then squeezing flow has been of great interest for various researchers [6–8].

Investigations with heat transport phenomenon are important in many industrial applications such as wire coating, hot rolling, metal spinning, glass blowing, paper manufacturing, glass fiber, glass sheet productions, the drawing of plastic films, continuous casting and the aerodynamic extrusion of plastic sheets. Radiative heat transfer plays an important role in controlling where convective heat transfer coefficients are small. In polymer industry, the radiative heat transfer is an essential phenomenon for the design of reliable equipments, nuclear plants, gas turbines, etc. The process of radiative heat transfer is also important in free and forced convection flows. Keeping all these applications in mind, many authors studied various research problems for the case of radiative heat transfer. Some of these can be seen in [9–12] and references therein.

High nonlinearity of governing equations in various flow problems means unlikeliness of exact solution. To cope up with these problems, many approximation techniques have been developed. Highly nonlinear problems such as the ones discussed above are therefore solved by using these techniques. These include, Adomian's Decomposition Method (ADM), Variational Iteration Method, Variation of Parameters Method, Homotopy Perturbation Method, Homotopy Analysis Method, etc. [13–20].

Mathematical models describing the realistic flow problems mostly involve the Non-Newtonian fluids. One of these fluids is known as Casson fluid and its formulation is provided in [21,22],

* Corresponding author.
E-mail address: syedtauseefs@hotmail.com (S.T. Mohyud-Din).

http://dx.doi.org/10.1016/j.ast.2015.10.019
1270-9638/© 2015 Elsevier Masson SAS. All rights reserved.

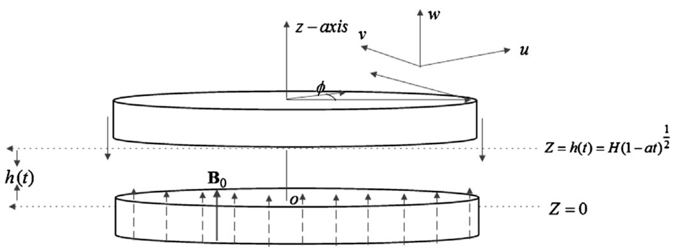

Fig. 1. Schematics diagram of the problem.

it is found to be suitable for blood flow problems up to large extent.

Casson fluid flow between squeezing disks under radiative heat transfer has not been considered yet. To fill out this gap, heat transfer analysis for squeezing flow of Casson fluid between parallel disks under the effects of thermal radiation is presented. Well known Homotopy Analysis Method (HAM) [23–36] is employed to solve the problem. Graphs are plotted to analyze the effects of different emerging parameters on velocity and temperature profiles.

2. Governing equations

We have investigated parallel infinite disks h distance apart with magnetic field practiced vertically and being proportional to $\boldsymbol{B_0}(\boldsymbol{1}-\boldsymbol{at})^{\frac{1}{2}}$ (Fig. 1). Incompressible Casson fluid is used in between the disks. Magnetic field is negligible for low Reynold numbers.

Consequently, Casson fluid flow equation is delineated as [31–34]

$$\tau_{ij}=\begin{Bmatrix} 2\left[\mu_B+\left(\dfrac{p_y}{2\pi}\right)\right]e_{ij}, & \pi>\pi_c \\ 2\left[\mu_B+\left(\dfrac{p_y}{2\pi_c}\right)\right]e_{ij}, & \pi_c>\pi \end{Bmatrix}$$

where p_y is yield stress of fluid, μ_B the plastic-dynamic viscosity of the fluid and π is the self-product of component of deformation rate with itself and π_c is the critical value of the said self-product. T_w and T_h are constant temperatures for lower and upper disk respectively. The viscous dissipation effects in the energy equation are retained. We have chosen the cylindrical coordinates system (r,ϕ,z), where the upper disk is moving with velocity $\frac{\boldsymbol{aH}(\boldsymbol{1}-\boldsymbol{at})^{-\frac{1}{2}}}{\boldsymbol{2}}$ towards or away from the stationary lower disk. Thus, the constitutive equations for two-dimensional flow and heat transfer of a viscous fluid under the effects of thermal radiation can be written as:

$$\frac{\partial\bar{u}}{\partial r}+\frac{\bar{u}}{r}+\frac{\partial\bar{w}}{\partial z}=0 \tag{1}$$

$$\rho\left(\frac{\partial\bar{u}}{\partial t}+\bar{u}\frac{\partial\bar{u}}{\partial r}+\bar{w}\frac{\partial\bar{u}}{\partial z}\right)=-\frac{\partial\hat{p}}{\partial r}+\mu\left(\frac{1+\beta}{\beta}\right)\left(\frac{\partial^2\bar{u}}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{u}}{\partial r}-\frac{\bar{u}}{r^2}+\frac{\partial^2\bar{u}}{\partial z^2}\right)-\frac{\sigma}{\rho}B^2(t)\bar{u}, \tag{2}$$

$$\rho\left(\frac{\partial\bar{w}}{\partial t}+\bar{u}\frac{\partial\bar{w}}{\partial r}+\bar{w}\frac{\partial\bar{w}}{\partial z}\right)=-\frac{\partial\hat{p}}{\partial z}+\mu\left(\frac{1+\beta}{\beta}\right)\left(\frac{\partial^2\bar{w}}{\partial r^2}+\frac{\partial^2\bar{w}}{\partial z^2}+\frac{1}{r}\frac{\partial\bar{w}}{\partial r}\right), \tag{3}$$

$$C_p\left(\frac{\partial T}{\partial t}+\bar{u}\frac{\partial T}{\partial r}+\bar{w}\frac{\partial T}{\partial z}\right)=\frac{k}{\rho}\left(\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)-\frac{1}{\rho}\frac{\partial}{\partial z}(q_r)+\nu\left(1+\frac{1}{\beta}\right)\begin{pmatrix}2\left(\dfrac{\partial u}{\partial r}\right)^2+2\left(\dfrac{\partial w}{\partial r}\right)^2+2\dfrac{u^2}{r^2}\\ +\left(\dfrac{\partial u}{\partial z}\right)^2+2\left(\dfrac{\partial w}{\partial z}\right)^2+2\left(\dfrac{\partial u}{\partial z}\right)\left(\dfrac{\partial w}{\partial r}\right)\end{pmatrix}. \tag{4}$$

Supporting conditions are

$$\{\bar{u}=0,\ \bar{w}=-w_0\}|_{z=0}$$
$$\left\{\bar{u}=0,\ \bar{w}=\frac{dh}{dt}\right\}\Big|_{z=h(t)} \tag{5}$$

$$T=T_w|_{z=0}$$
$$T=T_h|_{z=h(t)} \tag{6}$$

In the above equations $\bar{u}$ and $\bar{w}$ are the velocity components in the r- and z-directions respectively, while ρ is the density, viscosity μ, $\hat{p}$ the pressure, specific heat C_p, T the temperature, ν kinematic viscosity, k thermal conductivity, k^* mean absorption coefficient, Stefan Boltzmann constant σ^*, and w_0 is suction/injection velocity.

Substituting the following transformations [8]

$$\bar{u}=\frac{ar}{2(1-at)}\bar{F}'(\eta),\quad \bar{w}=-\frac{aH}{\sqrt{1-at}}\bar{F}(\eta),$$
$$B(t)=\frac{B_0}{\sqrt{1-at}},\quad \eta=\frac{z}{H\sqrt{1-at}},\quad \theta(\eta)=\frac{T-T_h}{T_w-T_h}, \tag{7}$$

in Eqs. (2)–(4) and removing the pressure gradient from the subsequent equations, we finally obtain

$$\left(1+\frac{1}{\beta}\right)\bar{F}^{(iv)}-S\left(\eta\bar{F}'''+3\bar{F}''-2\bar{F}\bar{F}'''\right)-M^2\tilde{F}''=0, \tag{8}$$

$$\left((1+Rd(1+(\theta_w-1)\theta)^3\theta'\right)'+S\,\mathrm{Pr}\left(2\bar{F}\bar{\theta}'-\eta\bar{\theta}'\right)+\left(1+\frac{1}{\beta}\right)\mathrm{Pr}\,Ec\left(\bar{F}''^2+12\delta^2\bar{F}'^2\right)=0, \tag{9}$$

with associated condition

$$\bar{F}(0)=A,\quad \bar{F}'(0)=0,\quad \theta(0)=1,$$
$$\bar{F}(1)=\frac{1}{2},\quad \bar{F}'(1)=0,\quad \theta(1)=0, \tag{10}$$

where A denotes the suction/injection, Eckert number Ec, Hartman number M, squeeze number S, Prandtl number Pr, radiation parameter R_d, θ_w is the temperature difference parameter, and δ the dimensionless length defined as

$$S=\frac{aH^2}{2\nu},\quad M^2=\frac{aB_0^2H^2}{\nu},\quad \mathrm{Pr}=\frac{\mu C_p}{K_0},\quad Rd=\frac{16\sigma^*T_h^3}{3k^*k},$$
$$Ec=\frac{1}{C_p(T_w-T_h)}\left(\frac{ar}{2(1-at)}\right)^2,\quad \delta^2=\frac{H^2(1-at)}{r^2}. \tag{11}$$

The skin friction coefficient and Nusselt number are defined as

$$C_{fr}=\frac{\tau_{rz}|_{z=h(t)}}{\rho\left(\frac{-aH}{2(1-at)}\right)^2},\quad Nu=\frac{H}{K_0(T_w-T_h)}(q_w+q_r), \tag{12}$$

where

$$\tau_{rz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\Big|_{z=h(t)},\quad q_w=-k_0\left(\frac{\partial T}{\partial z}\right)\Big|_{z=h(t)}. \tag{13}$$

In view of (7), Eq. (12) can be written as

Download English Version:

https://daneshyari.com/en/article/1717708

Download Persian Version:

https://daneshyari.com/article/1717708

Daneshyari.com