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Moment-independent importance measure of correlated input variable and its state dependent parameter solution

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ABSTRACT

For clearly exploring the origin of the uncertainty of structure failure probability in case the correlated input variables are involved, a new moment-independent importance measure of the input variable is firstly proposed on the failure probability. The intrinsic relationship between the proposed moment-independent importance measure and the corresponding variance-based importance measure is exposed. Then, based on the existing independent orthogonalization-based importance measures of the correlated input variables, the proposed moment-independent importance measure is further decomposed into two parts: the uncorrelated contribution due to the unique variations of a variable and the correlated one due to the variations of a variable correlated with other variables. Finally, combining the highly efficient state dependent parameter (SDP) method for the variance-based importance analysis of independent variables, a SDP solution is established for the moment-independent importance measure of correlated variables. Several examples are used to demonstrate that the proposed importance measures can effectively describe the effects of the input variables on the reliability of the structure, and the established solution can obtain the moment-independent importance measures efficiently for both the independent input variables and the correlated ones.

1. Introduction

Since 1960s, many researches have focused on the sensitivity analysis of the partial derivatives of structural responses, characters or indices with respect to input variables. However, those sensitivities are solved at nominal values, they cannot take account of the variation effect of the input variables, and thus those sensitivities are local. Compared with the local sensitivity, the importance measure, i.e. global sensitivity, is defined as that uncertainty in the output can be apportioned to different sources of uncertainty in the model input [1]. It is significant in engineering design and probability safety assessment, since it can comprehensively consider the average effect of the input variables on the output response in the value domain of the input variables. Thus, more and more studies nowadays are using importance analysis methods instead of local sensitivity analysis. Many importance analysis techniques are available. The nonparametric technique [2–4] is one alternative global sensitivity, but this method lacks the independence of the model. The variance-based importance analysis [5–8] is used extensively, but it implicitly assumes that the variance, i.e. the second moment, is sufficient to describe output variability [1]. However, any moment of a random variable provides a summary of its distribution with the inevitable "loss of resolution" that occurs when the information contained in the distribution is mapped into a single number [2]. To analyze the effect of input variable on the whole distribution of the output, many moment-independent importance measures [9–11] have been presented. So far, the moment-independent importance measure proposed by Borgonovo [11] is applied more extensively than the others. However, the most crucial and difficult problem in obtaining Borgonvo's importance measure is how to obtain the unconditional and conditional probability density functions (PDF) of the model output rapidly and properly. Additionally, in the reliability analysis, most small failure probability is related to tail distribution of the performance response function, and the effect of input variable on the distribution of the output is not equal to that on the failure probability [12]. Therefore, a moment-independent importance measure of the input variable on the failure probability is presented in Ref. [12] to represent the effect of input variable on the failure probability, and the measure can provide useful information for reliability design more directly than that on the distribution of the performance response.

However, most of the existing importance analysis techniques assume input variables' independence, and a few studies have fo-

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cused on the importance analysis of the correlated input variables, which is usually the common case in the engineering. The correlation of the input variables may affect the importance ranking of the variables dramatically, therefore, only the importance analysis methods by taking the correlation of the variables into consideration can reflect the effects of the input variables on the output response reasonably and correctly. In addition, for response models with correlated input variables, to explore the origin of the uncertainty of the output response clearly, the contribution of uncertainty to output response by an individual input variable should be divided into two parts: the uncorrelated contribution (by the uncorrelated variations, i.e. the unique variations of a variable which cannot be explained by any other parameters) and correlated contribution (by the correlated variations, i.e. variations of a variable which are correlated with other input variables) [13]. This is very important, since it can help engineers decide if they need to focus on the correlated variations among specific variables (if the correlated contribution dominates) or the variables themselves (if the uncorrelated contribution dominates). Based on this idea, two kinds of methods are proposed in Refs. [13] and [14] respectively to decompose the contributions by the correlated input variables to the variance of response. However, both of them cannot work well with response model with major nonlinearity [15].

To overcome the limitations in Refs. [13] and [14], a set of variance-based sensitivity indices is proposed in Ref. [15] on a specific orthogonalization of the inputs and ANOVA-representations of the model output to perform importance analysis of models with correlated inputs. These indices can not only reflect the uncorrelated and correlated contributions to the variance of response by correlated input variables comprehensively, but also support nonlinear models and nonlinear dependences. However, all the above methods for the decomposition of the contribution by the correlated input variables to the uncertainty of output response are based on the variance-based importance measures of input variables, thus with the inevitable "loss of resolution".

To comprehensively and effectively analyze the effects of input variables on the failure of structure in reliability engineering, a new moment-independent importance measure of the input variable on the failure probability is firstly proposed on Ref. [12]. Then, based on the existing independent orthogonalization-based importance measures of the correlated input variables in Ref. [15], the total contribution by the correlated input variable to the failure probability of structure is further decomposed into partial ones contributed by the correlated and uncorrelated variations of input variables. And a highly efficient SDP solution is also established to calculate the corresponding moment-independent importance measures of correlated input variables, which can save computational cost considerably. The proposed importance measures and the established method can provide more referential information for clearly exploring the origin of the uncertainty of structure failure in case the correlated input variables involved.

2. The moment-independent importance measure of input variable on failure probability

In this section, the moment-independent importance measure of input variable on failure probability is defined. Then, we use statistical inference theory to expose the relationship between the defined moment-independent importance measure and the variance-based importance measure.

2.1. The importance measure δ_I of the input variable on the failure probability

In the reliability model $y = g(x_1, x_2, \ldots, x_n)$ of the structure, where y is the model output, and $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ are input variables with uncertainty. Denote P_{f_y} as the unconditional failure probability, i.e. $P_{f_y} = P\{g(\boldsymbol{x}) \le 0\}$. When the input variable $\boldsymbol{x}_I$ is fixed at its realization, the conditional failure probability $P_{f_{y|\boldsymbol{x}_I}}$ can be obtained. Based on the idea of the moment-independent importance analysis, the importance measure η_I of input variable $\boldsymbol{x}_I$ on the failure probability in Ref. [12] is defined as:

$$\eta_I = \frac{1}{2}E_{\boldsymbol{x}_I}\left[|P_{f_y} - P_{f_{y|\boldsymbol{x}_I}}|\right] = \frac{1}{2}\int_{-\infty}^{+\infty} |P_{f_y} - P_{f_{y|\boldsymbol{x}_I}}| f_{\boldsymbol{x}_I}(\boldsymbol{x}_I) d\boldsymbol{x}_I \tag{1}$$

where $\boldsymbol{x}_I$ represents a random input variable x_i or a set of random input variables $(x_{i_1}, \ldots, x_{i_g})$ $(1 \le i_1 \le \cdots \le i_g \le n)$. $f_{\boldsymbol{x}_I}(\boldsymbol{x}_I)$ is the joint PDF of $\boldsymbol{x}_I$ and $E[\bullet]$ is the operator of expectation.

The importance measure η_I reflects the effect of the input variable $\boldsymbol{x}_I$ on the failure probability of the structure when $\boldsymbol{x}_I$ takes its realizations according to $f_{\boldsymbol{x}_I}(\boldsymbol{x}_I)$.

The absolute value in equation (1) is difficult to compute. Thus, in this paper it is equally transformed into square operation. Denote the equally transformed importance measure of the input variable $\boldsymbol{x}_I$ on the failure probability as δ_I, and it can be expressed as follows:

$$\delta_I = E_{\boldsymbol{x}_I}[P_{f_y} - P_{f_{y|\boldsymbol{x}_I}}]^2 = \int_{-\infty}^{+\infty} [P_{f_y} - P_{f_{y|\boldsymbol{x}_I}}]^2 f_{\boldsymbol{x}_I}(\boldsymbol{x}_I) d\boldsymbol{x}_I \tag{2}$$

The definition in equation (2) shows that importance measure δ_I can reflect the effect of the input variable $\boldsymbol{x}_I$ on the failure probability of the structure as effectively as importance measure η_I. However, the newly defined importance measure has significant meaning, which can be seen later in section 2.2.

2.2. The relationship of importance measure δ_I with variance-based importance measure

The exact expression of failure probability of the structure is the integration of the joint PDF $f_{\boldsymbol{x}}(x_1, x_2, \cdots, x_n)$ of the inputs $\boldsymbol{x}$ in the failure domain, and it can be rewritten into the expectation of the indicator function I_F of the failure domain as follows:

$$\begin{aligned} P_{f_y} &= \int \cdots \int_{g(\boldsymbol{x}) \le 0} f_{\boldsymbol{x}}(x_1, x_2, \cdots, x_n) dx_1 dx_2 \cdots dx_n \\ &= \int \cdots \int_{R^n} I_F f_{\boldsymbol{x}}(x_1, x_2, \cdots, x_n) dx_1 dx_2 \cdots dx_n \\ &= E[I_F] \end{aligned} \tag{3}$$

where $I_F = \begin{cases} 1 & g(\boldsymbol{x}) \le 0 \\ 0 & g(\boldsymbol{x}) > 0 \end{cases}$ is the indicator function of the failure domain. R^n is the n-dimensional variable space.

Similar to the unconditional failure probability above, the conditional failure probability of the structure while $\boldsymbol{x}_I$ fixed at certain realization can be written as follows:

$$P_{f_{y|\boldsymbol{x}_I}} = E[I_F|\boldsymbol{x}_I] \tag{4}$$

where $I_F|\boldsymbol{x}_I = \begin{cases} 1 & g_{\boldsymbol{x}_I}(\boldsymbol{x}) \le 0 \\ 0 & g_{\boldsymbol{x}_I}(\boldsymbol{x}) > 0 \end{cases}$ is the indicator function of the failure domain of the conditional performance response function $g_{\boldsymbol{x}_I}(\boldsymbol{x})$, which is conditional on $\boldsymbol{x}_I$ having certain values.

Substitute equations (3) and (4) into equation (2), the relationship of the importance measure δ_I with the corresponding variance-based importance measure can be obtained as following

$$\delta_I = E_{\boldsymbol{x}_I}\left\{\left[E(I_F) - E(I_F|\boldsymbol{x}_I)\right]^2\right\} = V\left[E(I_F|\boldsymbol{x}_I)\right] \tag{5}$$

where $V[\bullet]$ is the operator of the variance.