Aerospace Science and Technology 49 (2016) 225–230

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Convective heat transfer and MHD effects on two dimensional wall jet flow of a nanofluid with passive control model

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ARTICLE INFO

Article history:
Received 29 July 2015
Received in revised form 15 November 2015
Accepted 5 December 2015
Available online 11 December 2015

Keywords:
Jet flow
Boundary layer flow
Convective heat transfer
Nanofluids
Passive control condition
MHD

ABSTRACT

Due to increasing demand of efficient cooling and heating systems in the field of automotive, aerospace and process industries, the heat transfer technology is gaining importance for the desirable solutions. By keeping in view the requirement of the efficient cooling/heating systems we have considered the problem of two dimensional convective laminar wall jet flow of a viscous nanofluid. The effects of thermo-diffusion and Brownian motion have also been considered during the investigation along with the convective boundary condition. Passive control condition is utilized to control the concentration of nanoparticles at the surface. After using similarity solution the obtained system of equations is solved by employing a well-known effective numerical scheme Runge–Kutta–Fehlberg method. The variation in velocity, temperature and concentration profile due to Magnetic forces, Lewis number and Prandtl number is recorded. It has been observed that Lorentz forces are important to control the extra vibrations in the fluid flow. Due to a passive control boundary condition, we have recorded a negligible effect of Brownian motion parameter on the heat transfer rate.

1. Introduction

Recently, the study of convective flows of nanofluids is getting popular due to their enhanced thermal conductivities as compared to the conventional fluids like oil, water, ethylene glycol, etc. The field of Nanotechnology opened new dimensions for many technologies likecooling systems, power generation, biotechnology, medicine, treatment of diseases like cancer, transportation, military and security departments. Initially, Choi et al. [1] used nano/micro sized particles in traditional liquids and observed that the heat transfer rate becomes doubled. Bég et al. [2] investigated the transport of non-Newtonian nanofluids through porous medium in the presence of micro-organisms. Uddin et al. [3,4] represented model for radioactive hydromagnetic thermosolutal flow of nanofluids. A non-similar solution of free convective flow of nanofluids was carried out by Khan et al. [5] for the better understanding of flow of nanofluids with convective boundary condition. In [6–8] many researchers recorded their input in the form of numerical results highlighting the effectiveness of nanofluids for the thermal conduction and heat transfer.

Whenever a fluid strikes a surface at an angle of 90 degrees, it spreads over forming a thin layer. Glauert [9] was the first one to perceive this idea and describe the phenomena as wall jet flow. In order to study the flow properties analytically, he introduced similarity solution by using boundary layer theory. His work was extended by Merkin and Needham [10] by considering the case of wall motion and suction/injection through the walls. They remarked that the similarity solution stated by Glauert for wall jet flow is only obtainable if we also consider blowing/suction, while discussing the problem of moving walls in jet flows. The conclusion tagged by Merkin and Needham [10] was endorsed by Magyari and Keller [11]. They also recorded that for variant values of skin friction on the wall, we can get a family of solutions.

In literature the commonly presented models for nanofluids are dispersion model [12], homogeneous model [13] and Buongiorno model [14]. Of all the models, homogeneous model is widely preferred to extend the governing equations of conventional fluids to nanofluids due to its simplicity and compatibility. Subsequently, all the conventional heat transfer parameters used to measure the thermo-physical characteristics could also be used for nanofluids. The Said model is substantial for low concentration nanofluids because they are similar to Newtonian fluids. When the concentration grows higher, then this model is not suitable to represent the behavior of nanofluids.

Many researchers recorded the flow properties of nanofluids by considering the homogeneous model. Bachok et al. [15] concluded that the rate of heat transfer can be improved by adding nanoparticles to ordinary fluids. They used the geometry of rotating disk

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http://dx.doi.org/10.1016/j.ast.2015.12.008

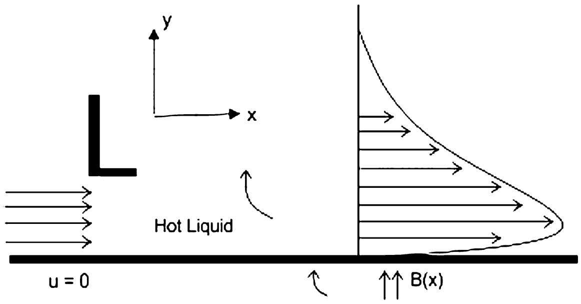

Fig. A. Geometry of Problem.

and investigated the heat transfer through boundary layer flow of a nanofluid. Various studies are available in literature using different models to analyze the nanofluids flow in different geometries. Some of these can be seen in [16–23] and references therein.

In this study, we consider the laminar wall jet flow of incompressible nanofluid spreading through a narrow vertical slit with convective and more realistic boundary conditions which were suggested by Kuznetsov and Nield [24,25]. Also it is recorded from the literature that Mele et al. [26] and Zhou et al. [27] investigated the stability of wall jet from laminar to turbulent state by giving the range of Reynold number from 300 to 500. The similarity solution of the boundary layer problem with the transversely applied magnetic field is used to obtain the desirable form of governing equations. The assumption has been made for the existence of similarity solution as $u \propto x^{-1/2}$ and the heat transfer coefficient vary inversely to the three-quarter root of the distance along the flat surface from the leading edge. Numerical approach has been used to analyze the correlated velocity, temperature and concentration profiles. Variation of different parameters is recorded and interpreted graphically to understand the insight of the problem. Effects of Hartman number, Biot number, Brownian motion parameter and thermophoresis parameter on Nusselt and Sherwood numbers have also been explored.

2. Mathematical modeling

We consider two dimensional fully developed incompressible laminar wall jet flow of a viscous nanofluid at temperature T_f through a vertical thin slit with convective flow over a plate at temperature T_w parallel to x-axis. The spread of fluid over the plate forms a boundary layer and at far field the temperature is represented by T_∞. Along with the convective boundary condition, a physically more practicable passive control model of the nanoparticle also taken into account. A transversely applied variable magnetic field $B(x)$ is also present as shown in Fig. A. The equations governing the flow under the aforesaid assumptions can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\upsilon_f\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma B^2(x)}{\rho_f}u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+\upsilon_f\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[D_B\left(\frac{\partial \phi}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left\{\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right\}\right], \tag{4}$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=D_B\left(\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right), \tag{5}$$

with boundary conditions

$$u=0,\quad v=0,\quad -k_f\frac{\partial T}{\partial y}=h_f(x)(T_f-T),$$
$$D_B\frac{\partial \phi}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0\quad \text{at } y=0,$$
$$u\to 0,\quad v\to 0,\quad T\to T_\infty,\quad \phi\to\phi_\infty,\quad \text{as } y\to\infty \tag{6}$$

In given boundary conditions the assumption of controlling the volume fraction of nanoparticle at the surface is discarded and instead of that a passive control model is utilized. According to which nanoparticle flux will be zero at the surface. Here, u and v are velocity components along x and y-axis, respectively, ρ_f is the density of base fluid, $\upsilon_f=\frac{\mu}{\rho_f}$ is kinematic viscosity, p is the pressure, σ is electrical conductivity and $B(x)$ is the perpendicularly applied variable magnetic field. Sheikholeslami et al. [28], Chiam et al. [29] and Sadoughi et al. [30] proposed to take $B(x)=B_0x^{\frac{r-1}{2}}$ where B_0 and r are physical constants. Also k_f is thermal conductivity of the fluid, α_f represents the thermal diffusivity, D_B is Brownian motion diffusion coefficient, D_T is thermophoresis diffusion coefficient, T is fluid temperature and ϕ is the nanoparticle volume fraction. $(\rho c)_f$ and $(\rho c)_p$ are the heat capacities of fluid and nanoparticle material respectively. Besides τ is the parameter defined by $\frac{(\rho c)_f}{(\rho c)_p}$. Using boundary layer approximations, Eqs. (1)–(5) reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B(x)^2}{\rho_f}u, \tag{8}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_f\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[D_B\left(\frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left\{+\left(\frac{\partial T}{\partial y}\right)^2\right\}\right], \tag{9}$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=D_B\left(\frac{\partial^2 \phi}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right). \tag{10}$$

To get inside analysis of the problem, we use following similarity transforms [9,31]

$$\eta=\frac{y}{\nu^{1/2}x^{3/4}},\quad \psi=4\sqrt{\nu}x^{1/4}f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},$$
$$\phi(\eta)=\frac{\phi-\phi_\infty}{\phi_\infty}\quad \text{(for passive control of } \phi) \tag{11}$$

We define stream function ψ as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$.

By applying Eq. (11), the Eq. (7) is identically satisfied and a system of differential equations is obtained from Eqs. (8)–(10) as follows,

$$f'''+ff''+2f'^2-M^2f'=0, \tag{12}$$

$$\theta''+\Pr f\theta'+\Pr Nb\theta'\phi'+\Pr Nt\theta'^2=0, \tag{13}$$

$$\phi''+Lef\phi'+\frac{Nt}{Nb}\theta''=0. \tag{14}$$

We get the reduced boundary conditions by utilizing Eq. (11),

$$f(0)=0,\quad f'(0)=0,\quad \theta'(0)=-b\big(1-\theta(0)\big),$$
$$Nb\phi'(0)+Nt\theta'(0)=0,$$
$$f(\infty)\to 1,\ \theta(\infty)\to 0,\ \phi(\infty)\to 0, \tag{16}$$