Aerospace Science and Technology 40 (2015) 47–55

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Analytic approach to determine optimal conditions for maximizing altitude of sounding rocket

Sang-Hyeon Lee [1]

Dept. of Mechanical Engineering, University of Ulsan, Ulsan, 680-749, Republic of Korea

ARTICLE INFO

Article history:
Received 30 May 2014
Received in revised form 5 August 2014
Accepted 27 October 2014
Available online 31 October 2014

Keywords:
Sounding rocket
Rocket momentum equation
Analytic solution
Optimal conditions for maximizing altitude

ABSTRACT

An analytic approach to determine the optimal conditions for maximizing altitude of a sounding rocket at burn-out state or stationary state is suggested. The one-dimensional rocket momentum equation including thrust, gravitational force, and aerodynamic drag is solved. The typical case that has an analytic solution of the rocket equation is considered. Also, numerical calculations are conducted for comparisons. To build a standard approach to determine the optimal conditions, flights in a constant atmosphere where the air density is constant are considered. The analytic solutions agree well with the numerical ones. It is shown that the optimal condition for maximizing altitude at burn-out state or stationary state can be predicted with a characteristic equation.

© 2014 Elsevier Masson SAS. All rights reserved.

1. Introduction

Sounding rockets carry scientific instruments to altitudes from 50 km to more than 300 km to explore the atmosphere at high altitude or near outer space. Scientific studies with a sounding rocket are simple, fast, and inexpensive compared to those with a satellite. The costs for a sounding rocket mission are much lower than those required for an orbiter mission, since sounding rocket missions do not need expensive boosters, extended telemetry or tracking coverage. Mission costs are also low because of the acceptance of a higher degree of risk in the mission compared to orbital missions [13]. Many countries are running sounding rocket programs and trying to develop technologies related to sounding rockets to exploit these advantages [1,17,10,5,12,7,3,8,18,2,16]. Most scientific measurements, observations, or experiments for sounding rocket missions are carried out near apogee. The low speed near apogee provides favorable chances to explore or observe space in a short time period. Furthermore, there are some important regions of space that are too low to be sampled by satellites; thus, sounding rockets provide platforms to carry out in-situ measurements in these regions [8]. Some missions are carried out during free-fall that provides microgravity environments [15,21] or good hypersonic condition at low cost [14,19].

Sounding rockets usually have parabolic trajectories during the whole flight range in order to secure safety of launching site or citizens and to collect used rocket bodies or parts. In parabolic motions, there are no accelerations or body forces in the lateral direction, which makes it simple to analyze the lateral motion of a rocket. Therefore, in the present study, we consider vertical direction only, for simplicity. The motion of a rocket can then be described with a one-dimensional momentum equation that includes thrust, gravitational force, and aerodynamic drag force.

The rocket mass varies with time, and the drag force is proportional to the square of the rocket velocity, which makes the governing equation nonlinear. Thus, we cannot obtain analytic solutions in a general form. Hence in most cases, numerical approaches are used to obtain solutions of the rocket equation because of easy implementation with less assumptions. Analytic solutions, on the contrary to numerical ones, are exact without numerical errors, give insights to understand the behavior of the system, show the critical parameters, and lead to ways to determine the optimal conditions. Therefore it is necessary to find out analytic solutions if possible. An approximate analytic solution can be obtained with neglecting drag force but it contains serious errors especially near ground. There are also approximate solutions with the Taylor series expansion, the perturbation method or the least square method [9], but they are complex and unhandy. An analytic exact solution of the rocket equation including drag force exists only in a typical situation where the three forces are well balanced. In the present study, we consider the typical cases where analytic exact solutions exist.

The design target of a sounding rocket is the altitude. The ejection conditions of propellant jet determine thrust force, rocket velocity or boost time that eventually change the rocket altitude at burn-out state or at stationary state. Therefore, it is necessary to determine an optimal jet condition for maximizing the altitude at

E-mail address: lsh@mail.ulsan.ac.kr.

[1] Professor, Div. Aerospace Engineering,

http://dx.doi.org/10.1016/j.ast.2014.10.012
1270-9638/© 2014 Elsevier Masson SAS. All rights reserved.

Nomenclature

g	gravitational acceleration
h	altitude
K	drag parameter
m	rocket mass
$\dot{m}$	rate of rocket mass change or mass flow rate of propellant jet
q	velocity parameter for rocket velocity
t	time
u_e	velocity of propellant jet
v	rocket vertical velocity
p	static pressure
ρ	density

Subscripts

b	burn-out state
o	ground state
opt	optimal condition for maximizing altitude
s	stationary state

given launching conditions. Despite the wide usage of sounding rockets, as far as we know, there are no analytic methods that are simple and convenient enough to determine the optimal condition for maximizing altitude. Hence, the objective of the present study is to suggest an analytic method to determine the optimal condition for maximizing altitude at burn-out state or stationary state.

As a beginning study, we first consider the flight in a constant atmosphere where the air density is constant, since variable air density makes it impossible to obtain analytic solutions valid through whole flight range. In the standard atmosphere, the air density diminishes and thus the aerodynamic drag decreases as the altitude increases, which means that assuming constant air density leads to considerable underestimations of the altitude. However, even though limited to the flight in a constant atmosphere, the approach in the present study will show a standard procedure to obtain analytic solutions and to determine optimal conditions. Also, useful bases would be built for developing methods to solve the governing equation for the fights in a real atmosphere.

2. One-dimensional rocket equation

2.1. Equation in boost phase

The motion of a rocket in boost phase climbing in the vertical direction can be described with the following one-dimensional rocket equation [20,6].

$$m\frac{dv}{dt} = F - D - mg. \tag{2.1}$$

The mass of a rocket decreases with the mass flow of propellant.

$$m = m_o + \int_o^t \dot{m}dt, \tag{2.2a}$$

$$\dot{m} = \frac{dm}{dt}. \tag{2.2b}$$

The mass flow rate $\dot{m}$ is equal to the rate of the rocket mass and has a negative sign by definition.

The terms F and D stand for thrust force and drag force, respectively. The rocket thrust is composed of two parts:

$$F = \dot{m}u_e + A_e(p_e - p_a). \tag{2.3}$$

The term A_e is the area at the exit plane of a rocket nozzle and the term p_a is the ambient air pressure. For an adiabatic nozzle flow, the total enthalpy is constant, and then we can assume that the jet velocity u_e is constant. The jet velocity has the negative sign since its direction is opposite to the rocket velocity; thus, the thrust term $\dot{m}u_e$ has the positive sign. If the nozzle flow has a perfect expansion, the second term of the thrust vanishes. Hereafter, we ignore the second term of the thrust for simplicity.

The aerodynamic drag force D can be represented with the drag parameter K as follows:

$$D = Kv^2, \tag{2.4a}$$

$$K = \frac{S}{2}C_d\rho_a. \tag{2.4b}$$

The terms S, C_d and ρ_a denote cross-section area of a rocket, aerodynamic drag coefficient and ambient air density, respectively. In the present study, we consider the aerodynamic drag coefficient as a constant for simplicity. Then, for a constant atmosphere where the ambient air density is constant, we can treat the drag parameter K as a constant.

Then the governing equation becomes

$$m\frac{dv}{dt} = \dot{m}u_e - Kv^2 - mg. \tag{2.5}$$

The mass is variable with time, and the square of the solution appears in the drag force, which makes the governing equation nonlinear. Thus, we cannot obtain analytic solutions in a general form. However, there is a typical case where an analytic solution exists. We introduce a velocity parameter as follows:

$$q = \sqrt{\frac{\dot{m}u_e - mg}{K}}. \tag{2.6}$$

The governing equation can then be reduced as

$$m\frac{dv}{dt} = K\left(q^2 - v^2\right). \tag{2.7a}$$

Separating variables leads to

$$\frac{dv}{q^2 - v^2} = \frac{K}{m}dt. \tag{2.7b}$$

This equation can be represented according to the mass instead of the time as follows:

$$\frac{dv}{q^2 - v^2} = \frac{K}{m}\frac{dm}{\dot{m}}. \tag{2.7c}$$

If the velocity parameter is constant, the left hand side can be analytically integrated, yielding an analytic solution of the governing equation if the term $K/m\dot{m}$ is analytically integrated.

2.2. Equation in coast phase

After the propellant is totally consumed, the flight phase turns into coast phase, where a rocket climbs with inert force until stationary state or apogee. The mass flow of propellant jet disappears in the equation and the rocket mass is kept constant. Also the velocity parameter shown in boost phase is irrelevant in coast phase. Then the rocket equation becomes

$$m_b\frac{dv}{dt} = -Kv^2 - m_b g. \tag{2.8}$$

Download English Version:

https://daneshyari.com/en/article/1717761

Download Persian Version:

https://daneshyari.com/article/1717761

Daneshyari.com