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Short communication

Suboptimal continuous torque attitude maneuvers for underactuated spacecraft

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ABSTRACT

This work provides a suboptimal continuous torque solution for controlling an underactuated spacecraft using two control inputs. A sequential submaneuver strategy is proposed. Classical quadratic penalty approaches for the control lead to jump discontinuities in the resulting control time history that can potentially excite an unwanted flexible body response. To address this operational concern, this work revises the definition for the optimal control problem by introducing a torque-rate penalty in the performance index. The significant advantage derived from this approach is that control design freedom becomes available for specifying internal control state boundary conditions, which effectively eliminates the undesirable jump discontinuities. Further optimization is achieved by introducing a positive definite weight matrix that penalizes the quadratic control term. This weight matrix allows optimal tuning for the control performance where a balance is achieved between the often conflicting goals of minimizing the applied control during the maneuver and simultaneously enforcing the requirement that the resulting control be free of jump discontinuities. Numerical simulation results are presented to prove the effectiveness of the proposed method. The resulting control solutions are compared with a related approach for an underactuated system.

1. Introduction

This work presents a generalization of recent work by Kim and Turner [12] where they present a suboptimal control solution for an underactuated spacecraft. Many references for underactuated system control are listed in Ref. [12], thus a brief summary is provided. Most authors assume an axis-symmetric rigid body to simplify the underactuated system control problem. Krishnan et al. [17] and Tsiotras et al. [23] have treated an attitude stabilization problem of a rigid spacecraft using two gas jet actuators. Tsiotras [22] has presented a partial solution to the problem of optimal feedback reorientation, and Shen and Tsiotras [21] have treated a minimum-time reorientation problem. Tsiotras and Luo [25] have provided stabilizing feedback control laws for the kinematic system of an underactuated spacecraft subject to input constraints. In addition, several time-invariant and time-varying control laws are introduced in Refs. [2,18,24,26]. Bacconi et al. [1] have proposed a switching logic based control law, and Hall et al. [7] have presented a control design, which combines a generalized inverse component for feedback linearization and an auxiliary input. Using the advantages of control moment gyros, angular rate damping and attitude control problems are handled in Refs. [8,29], respectively. For an asymmetric rigid spacecraft, Kim et al. [14,15] have presented a Davidenko-like homotopy method that generates a nonlinear open-loop solution for three-dimensional maneuvers when only two actuators are available. A sequential maneuver strategy is introduced by Kim et al. in Ref. [16] to avoid rotation about a failed actuator direction. This work is extended by Kim et al. in Ref. [13] where an optimal control problem formulation is introduced.

The main contribution of this work is that the resulting control time histories are free of discontinuous changes initially, finally, and during the maneuver. This design freedom is accomplished by adding a torque-rate penalty to the torque-minimizing performance index with a positive definite weight matrix. The weight matrix serves as a key between penalizing the magnitude of torque consumption and the smoothness of control. This approach enables the underactuated system to generate suboptimal control profiles that are free of jump discontinuities. It is desirable to eliminate control jump discontinuities because they can excite the higher mode flexural degrees of freedom because of the high frequency

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content of the control profiles [4–6,27]. By eliminating jump discontinuities, this work seeks to minimize spill-over to the response of the flexural degrees of freedom.

This work is organized as follows: (i) general rotational dynamics are slightly modified for describing the case where an actuator fails among three actuators, (ii) the modified Rodrigues parameters (MRPs) are introduced as an attitude representation for any large-angle maneuver descriptions except for a complete revolution, (iii) necessary conditions are developed using the torque-rate appended form of the performance index, (iv) switch-time boundary conditions are formulated, and (v) simulation results are presented to compare control profiles according to the torque-rate penalty portion for the performance index.

2. Problem formulation

2.1. Dynamics and kinematics

The method is presented for the special case of a control actuator failure about the 3rd principal body axis, though the methodology easily generalizes to handle control failures about any body axis. To describe a case for the failure of an actuator, which is aligned with the 3rd principal body axis, the rotational dynamics equation of a rigid body [10,11,28] is slightly modified as

$$\dot{\boldsymbol{\omega}} \triangleq \boldsymbol{p}(\boldsymbol{\omega}, \boldsymbol{u}) = [J]^{-1}\left(-\left[\boldsymbol{\omega}^{\times}\right][J]\boldsymbol{\omega} + [P]\boldsymbol{u}\right) \tag{1}$$

where $[J] \in \mathcal{R}^{3\times 3}$ is the positive definite inertia tensor for the spacecraft, $\boldsymbol{\omega} \in \mathcal{R}^3$ is the angular velocity vector of the spacecraft, and $\boldsymbol{u} \in \mathcal{R}^2$ is the available control torque vector. The control mapping matrix, $[P] \in \mathcal{R}^{3\times 2}$, and the cross product of the generic variable ($\boldsymbol{d}$), $[\boldsymbol{d}^{\times}] \in \mathcal{R}^{3\times 3}$, are defined as

$$[P] \triangleq \begin{bmatrix} I_{2\times 2} \\ \mathbf{0}_{1\times 2} \end{bmatrix}, \qquad \left[\boldsymbol{d}^{\times}\right] \triangleq \begin{bmatrix} 0 & -d_3 & d_2 \\ d_3 & 0 & -d_1 \\ -d_2 & d_1 & 0 \end{bmatrix}$$

The MRPs are a set of coordinates, which provide any attitude description except for a complete revolution [20]. The MRPs are defined in terms of the quaternion $(\boldsymbol{\rho}, q_4)$ or the principal rotational elements $(\Phi, \hat{\boldsymbol{e}})$ as

$$\boldsymbol{\sigma} = \frac{\boldsymbol{\rho}}{1+q_4} = \hat{\boldsymbol{e}} \tan\frac{\Phi}{4} \tag{2}$$

where the MRPs have a geometric singularity at $\Phi = \pm 2\pi$ from Eq. (2).

The governing kinematic differential equation for the MRPs is given by

$$\dot{\boldsymbol{\sigma}} \triangleq \boldsymbol{r}(\boldsymbol{\sigma}, \boldsymbol{\omega}) = \frac{1}{4}\left[B(\boldsymbol{\sigma})\right]\boldsymbol{\omega} \tag{3}$$

where $[B(\boldsymbol{\sigma})] \in \mathcal{R}^{3\times 3}$ is defined as

$$\left[B(\boldsymbol{\sigma})\right] \triangleq \left(1 - \boldsymbol{\sigma}^T\boldsymbol{\sigma}\right)[I_{3\times 3}] + 2\left[\boldsymbol{\sigma}^{\times}\right] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T$$

2.2. Optimal control formulation

In Ref. [12], the following quadratic torque penalty term is introduced in the performance index:

$$\mathcal{J} \triangleq \frac{1}{2}\int_{t_0}^{t_f} \boldsymbol{u}^T\boldsymbol{u}\,\mathrm{d}t \tag{4}$$

The optimal control solution for this problem is defined by a linear function of time. As a result, discontinuous control profiles are generated. This discontinuous control system behavior leads an unplanned excitation of the higher mode flexural degrees of freedom because of the high frequency content of the control torque history. In addition, the resulting control profiles are relatively sensitive to modeling error and may therefore prove difficult to implement [9,11].

In order to eliminate mathematically generated discontinuities in control profiles, this work invokes the following performance index:

$$\mathcal{J} \triangleq \frac{1}{2}\int_{t_0}^{t_f} \left(\boldsymbol{u}^T[W]\boldsymbol{u} + \dot{\boldsymbol{u}}^T\dot{\boldsymbol{u}}\right)\mathrm{d}t \tag{5}$$

where $[W] \in \mathcal{R}^{2\times 2}$ is the diagonal positive definite weight matrix; and t_0 and t_f are fixed initial and final times, respectively; $\boldsymbol{u}$ is assumed to have two continuous time derivatives, and the torque-rate, $\dot{\boldsymbol{u}}$, is defined as

$$\dot{\boldsymbol{u}} \triangleq \boldsymbol{g}(\dot{\boldsymbol{u}}) = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{u} \tag{6}$$

In Eq. (5), the weight matrix permits a trade-off between penalizing the magnitude of torque consumption and the smoothness of control.

Previously, Kim and Turner proposed a sequential maneuver strategy [12] and it is generalized to account for uniformly smooth control profiles. The details for the sequential maneuver strategy are not discussed here (see Ref. [12]).

A solution for Eqs. (1), (3), and (6) must satisfy the following boundary conditions:

$$\begin{cases} \boldsymbol{\sigma}(t_0) = \boldsymbol{\sigma}_{t_0}, & \boldsymbol{\omega}(t_0) = \boldsymbol{\omega}_{t_0}, & \boldsymbol{u}(t_0) = \boldsymbol{u}_{t_0} \\ \boldsymbol{\sigma}(t_f) = \boldsymbol{\sigma}_{t_f}, & \boldsymbol{\omega}(t_f) = \boldsymbol{\omega}_{t_f}, & \boldsymbol{u}(t_f) = \boldsymbol{u}_{t_f} \end{cases} \tag{7}$$

where Eq. (7) describes the attitude, angular velocity, and control torque at the initial and final times. *Note that one has freedom to specify the boundary conditions for the control torque.*

2.2.1. Necessary conditions

Defining the Hamiltonian for the system

$$\mathcal{H} = \frac{1}{2}\left(\boldsymbol{u}^T[W]\boldsymbol{u} + \dot{\boldsymbol{u}}^T\dot{\boldsymbol{u}}\right) + \boldsymbol{\xi}^T\boldsymbol{r} + \boldsymbol{\mu}^T\boldsymbol{p} + \boldsymbol{\eta}^T\boldsymbol{g} \tag{8}$$

where the Lagrange multipliers associated with the MRPs, angular velocity, and control torque are $\boldsymbol{\xi} \in \mathcal{R}^3$, $\boldsymbol{\mu} \in \mathcal{R}^3$, and $\boldsymbol{\eta} \in \mathcal{R}^2$, respectively. The first-order necessary conditions are obtained as follows:

(i) State equations:

$$\dot{\boldsymbol{\sigma}} = \frac{\partial\mathcal{H}}{\partial\boldsymbol{\xi}}, \qquad \dot{\boldsymbol{\omega}} = \frac{\partial\mathcal{H}}{\partial\boldsymbol{\mu}}, \qquad \dot{\boldsymbol{u}} = \frac{\partial\mathcal{H}}{\partial\boldsymbol{\eta}} \tag{9}$$

(ii) Costate equations:

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= -\frac{\partial\mathcal{H}}{\partial\boldsymbol{\sigma}} = -\frac{1}{2}\left[\Lambda(\boldsymbol{\sigma}, \boldsymbol{\omega})\right]^T\boldsymbol{\xi}, \\ \dot{\boldsymbol{\mu}} &= -\frac{\partial\mathcal{H}}{\partial\boldsymbol{\omega}} = -\frac{1}{4}\left[B(\boldsymbol{\sigma})\right]^T\boldsymbol{\xi} - \left[\Sigma(\boldsymbol{\omega}, J)\right]^T\boldsymbol{\mu}, \\ \dot{\boldsymbol{u}} &= -\frac{\partial\mathcal{H}}{\partial\boldsymbol{u}} = -[W]\boldsymbol{u} - [P]^T[J]^{-1}\boldsymbol{\mu} \end{aligned} \tag{10}$$

where

$$\left[\Lambda(\boldsymbol{\sigma}, \boldsymbol{\omega})\right] \triangleq \begin{bmatrix} \sigma_1\omega_1 + \sigma_2\omega_2 + \sigma_3\omega_3 & \omega_3 - \sigma_2\omega_1 + \sigma_1\omega_2 & \sigma_1\omega_3 - \sigma_3\omega_1 - \omega_2 \\ \sigma_2\omega_1 - \omega_3 - \sigma_1\omega_2 & \sigma_1\omega_1 + \sigma_2\omega_2 + \sigma_3\omega_3 & \omega_1 - \sigma_3\omega_2 + \sigma_2\omega_3 \\ -\sigma_1\omega_3 + \sigma_3\omega_1 + \omega_2 & \sigma_3\omega_2 - \omega_1 - \sigma_2\omega_3 & \sigma_1\omega_1 + \sigma_2\omega_2 + \sigma_3\omega_3 \end{bmatrix},$$

$$\left[\Sigma(\boldsymbol{\omega}, J)\right] \triangleq \begin{bmatrix} 0 & \frac{J_2 - J_3}{J_1}\omega_3 & \frac{J_2 - J_3}{J_1}\omega_2 \\ \frac{J_3 - J_1}{J_2}\omega_3 & 0 & \frac{J_3 - J_1}{J_2}\omega_1 \\ \frac{J_1 - J_2}{J_3}\omega_2 & \frac{J_1 - J_2}{J_3}\omega_1 & 0 \end{bmatrix}$$