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Analytical design methods for determining Moon-to-Earth trajectories

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ABSTRACT

To support China's lunar exploration mission requirements of high-latitude landing and anytime return, i.e., the capability of safely returning the crew exploration vehicle at any time from any lunar parking orbit, an analytical model for determining a transearth trajectory is presented. With a finite sphere of influence model, families of analytical Moon-to-Earth return trajectories are generated and analyzed to observe the characteristics in their Moon departures and Earth encounters. The requirement of high-latitude landing for the return phase trajectory is considered in the modified analytical model. No initial guess is required to generate the analytical solution. The results presented here are limited to a single impulsive maneuver. The difference between the results of the analytical model and a high-fidelity model is compared. This difference is relatively small and can be easily eliminated by a simple differential correction procedure. The solution can be used to establish the orbital launch window for Moon-to-Earth return and to serve as an initial estimate for future optimization procedures.

1. Introduction

Over the last decade, a number of lunar exploration missions have been launched or executed as means to realize China's manned lunar landing mission between 2025 and 2030. This exploration program, undertaken by the China National Space Administration (CNSA), is divided into three main operational phases [11]. The first phase of the program entails the launches of two lunar orbiters, Chang'e 1 and 2. The principal objectives are to transmit high-resolution images of the lunar surface to aid in the selection of a future landing site for the Chang'e 3 lander and rover mission and to test the key technology required for a soft-landing on the Moon. The second and the final phases of the exploration program are a soft landing and an automated sample return in 2017, as originally planned. As part of the requirements for a manned lunar landing, the spacecraft must have the capability to safely return from the Moon to the Earth at any time and from any lunar parking orbit in case the mission has to be aborted. In addition, due to the particular aims of China lunar mission third phase requirement, the construction of the Moon-to-Earth return trajectory must meet the requirements of a high-latitude landing, such as at Siziwangqi in Inner Mongolia [11]. For such a landing, it has been indicated that a better performance of the heat shield of the command module is required.

The early work examining Moon-to-Earth return trajectories was motivated by the requirement of safely returning a human crew to the Earth [3]. Miele [16] developed the theorem of image trajectories in the Earth–Moon space within the framework of the restricted three-body problem. The relationship between the outgoing/returning trajectories and the formulated theorem of image trajectories for feasible paths was established. Lancaster et al. [14] applied the matched asymptotic expansion method to the guidance problem of aborting from a specific lunar orbit and returning to the Earth while adhering to prescribed constraints. The optimization of the abort maneuver with a single impulse was also considered. Baoyin [2] investigated the ballistic trajectories to and from the vicinity of the Lagrange points L_1 and L_2 and the surfaces of the primaries in the restricted three-body problem. Such trajectories will be used in sample return missions and future crewed missions. Ikawa [12] derived a coplanar, three-body trans-Earth-lunar and return trajectory simulation methodology. The case for a spacecraft with a single impulse was investigated to find a reachable domain [24]. Dallas [4] studied the general characteristics of Moon-to-Earth return trajectories by applying the matched conic theory. An analysis of all types of Moon-to-Earth return trajectories demonstrates that the counterclockwise class of trajectories is superior to the clockwise class of trajectories; the exit point of the outgoing asymptote of the selenocentric hyperbola for a constant reentry path angle, the Earth-to-Moon distance, and the time of flight form a circular locus at the lunar sphere of influence (LSOI) with the exit point of the vertical impact outgoing asymptote of

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the selenocentric hyperbola as the center, and for each exit point on the locus of exit points there is a corresponding impact point on the locus of impact points. Less discussion has been devoted to the generation of transearth trajectories.

Unlike earlier Apollo missions to the Moon, NASA's currently proposed lunar mission is much more technologically demanding. For example, the spacecraft must have the capability to safely return a human crew to the Earth at any time and from any lunar parking orbit in case the mission has to be aborted [20,10]. The characteristics of orbits around the Moon and other celestial body have been also analyzed extensively [1,15]. Robinson and Geller [20] developed a simple and robust targeting algorithm for lunar transearth injection (TEI), intending for its onboard use during contingency and abort scenarios. Ocampo and Saudemont [18] examined the any-time abort capability required for human lunar missions. Again, an analytical procedure was used to generate a time-free initial estimate for a complete Moon-to-Earth return trajectory, which was then corrected using a differential correction procedure with the high-fidelity model. Yan and Gong et al. [25] used the Bellman pseudospectral method to generate high-accuracy solutions for high-fidelity trajectory optimization problems. This method rejects the propagation error stemming from control implementation and results in a finite-thrust, high-accuracy optimal Moon-to-Earth transfer trajectory. Miele and Mancuso [17] presented a systematic study of the optimization processes for Earth–Moon–Earth trajectories. The problem was formulated with a simplified version of the restricted three-body model and was solved with the sequential gradient-restoration algorithm for mathematical programming problems. This aforementioned work mainly concentrated on solving a two-point boundary value problem [13] to generate a first approximation of the desired Moon–Earth or Earth–Moon trajectory, which would then be input into a high-fidelity model as an initial estimate to search for the best parameters that satisfy the set of terminal conditions.

In response to these new challenges, finite-burn techniques have been developed to optimize escape trajectories that build upon impulsive maneuver models [25,9]. Weeks et al. [23] discussed the adaptation and implementation of a modified two-level correction process as the onboard targeting algorithm for the TEI phase of the Crew Exploration Vehicle (CEV). In their work, the magnitude of the thrust is assumed to be constant, and the thrust steering rate is constant or zero. Unlike earlier Apollo missions to the Moon, project Orion intends to land near the Polar Regions, and to reduce the fuel expenditure associated with the plane-changes prior to the Earth return, a three-maneuver sequence is employed during the return phase. Park and Gong [19] presented a time-bounded fuel-optimal Moon-to-Earth trajectory design for manned lunar missions using a restricted four-body model and included the gravitational effects of the Sun, the Moon and the Earth. The resulting optimal control problem was solved using the Legendre pseudospectral method. Fazelzadeh and Varzandian [6] obtained the minimum-time orbital trajectories for the Earth-to-Moon and Moon-to-Earth flights of continuous-thrust spacecraft by employing the time-domain finite element method. Numerical simulations highlighting the effects of the effective exhaust velocity parameter on the shape of the flight trajectory and the time of flight were also presented.

In this paper, the patch-conic model is incorporated as a precursor to the high-fidelity Earth–Moon gravitational field. For the Earth landing point always occurs in the vicinity that is associated with negative values of both the lunar declination and right ascension at the time of TEI, the apse line of the Earth-focused elliptical trajectory should be offset to satisfy the lunar mission third phase requirement for a high-latitude landing. The analytical models of the lunar probe transfer trajectories are developed to investigate the characteristics. First, the gravitational model upon which the described analysis and computer program are based is presented. The patched-conic technique is used with a finite sphere of influence model to generate the transearth trajectories under the set of boundary conditions. In addition, to completely satisfy the human mission requirements, a modified analytical trajectory model is introduced. In addition, this analytical trajectory model can be used to establish the orbital launch window, which is not discussed here but constitutes part of our current research program. Finally, a high-fidelity multibody gravitational model that includes the effects of the oblateness of the Earth, the solar radiation pressure and the third body's gravity is used, and a differential correction procedure is employed to eliminate the effect of these perturbations.

2. The gravitational model

The model upon which the following analysis and computer program are based was first used by Egorov [5] in 1956 and has since been successfully applied at the Space Technology Laboratories. This model assumes that the gravitational field of the Earth–Moon system consists of two independent inverse-square force fields, one associated with the Earth and the other associated with the Moon. While the spacecraft is inside the sphere of influence of the Moon (LSOI), only the Moon's gravity is considered. Outside of the LSOI, the only effect considered is the Earth's gravitational field. The radius of the LSOI is typically defined as

$$r_{soi} \approx 0.87 r_{EM} \left(\frac{m_M}{m_E} \right)^{\frac{2}{5}} \tag{1}$$

where m_M and m_E are the masses of Moon and Earth, respectively, and r_{EM} is the distance from the Moon to the Earth.

Under this system, the Moon-to-Earth trajectory consists of two Keplerian orbits, usually hyperbolic in the LSOI and elliptic in the sphere of influence of the Earth, which are "patched" into the LSOI to make the trajectory continuous. Thus, the patched-conic technique relates the characteristics of the selenocentric approach hyperbola to the characteristics of transearth trajectory [21]. To ensure that the trajectory is continuous across the LSOI, it is necessary to perform the following translations

$$\mathbf{r}_{Mc} = \mathbf{r}_{Ec} - \mathbf{r}_{EM} \tag{2}$$

$$\mathbf{v}_{Mc} = \mathbf{v}_{Ec} - \mathbf{v}_{EM} \tag{3}$$

where $(\mathbf{r}_{Mc}, \mathbf{v}_{Mc})$ and $(\mathbf{r}_{Ec}, \mathbf{v}_{Ec})$ are the position and velocity vectors of the spacecraft with respect to the Moon and Earth, respectively, at the exit point of the LSOI, henceforth referred to as c. $(\mathbf{r}_{EM}, \mathbf{v}_{EM})$ are the position and velocity vectors of the Moon, respectively, with respect to the Earth at the time that the spacecraft passes through point c. To obtain general analytical information about the relationships between the TEI conditions and the geocentric orbital parameters, some approximations are required. In this study, the Moon is assumed to move in an elliptical orbit defined as follows: setting the state of the Moon on Jan 1, 2025, which is obtained from the JPL DE405 ephemeris, as the initial conditions that are then propagated forward in time under the Earth's gravitational field only, hence obtaining the analytical model used for determining the lunar orbit.

3. Transearth trajectory generation

An analytical model has been developed using the patched-conic technique to produce transearth trajectories that satisfy specific China lunar mission third phase requirements. Under the double two-body gravitational model proposed by Egorov, the transearth trajectory can be represented by two conic sections,