Aerospace Science and Technology 45 (2015) 17–29

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Multi-fidelity robust aerodynamic design optimization under mixed uncertainty

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ARTICLE INFO

Article history:
Received 6 February 2015
Received in revised form 15 April 2015
Accepted 19 April 2015
Available online 22 April 2015

Keywords:
Multi-fidelity modeling
Polynomial chaos
Computational fluid dynamics
Aerodynamic shape optimization
Robust design
Output space mapping

ABSTRACT

The objective of this paper is to present a robust optimization algorithm for computationally efficient airfoil design under mixed (inherent and epistemic) uncertainty using a multi-fidelity approach. This algorithm exploits stochastic expansions derived from the Non-Intrusive Polynomial Chaos (NIPC) technique to create surrogate models utilized in the optimization process. A combined NIPC expansion approach is used, where both the design and the mixed uncertain parameters are the independent variables of the surrogate model. To reduce the computational cost, the high-fidelity Computational Fluid Dynamics (CFD) model is replaced by a suitably corrected low-fidelity one, the latter being evaluated using the same CFD solver but with a coarser mesh. The model correction is implemented to the low-fidelity CFD solutions utilized for the construction of stochastic surrogate by using multi-point Output Space Mapping (OSM) technique. The proposed algorithm is applied to the design of NACA 4-digit airfoils with four deterministic design variables (the airfoil shape parameters and the angle of attack), one aleatory uncertain variable (the Mach number) and one epistemic variable (β, a geometry parameter) to demonstrate robust optimization under mixed uncertainties. In terms of computational cost, the proposed technique outperforms the conventional approach that exclusively uses the high-fidelity model to create the surrogates. The design cost reduces to only 34 equivalent high-fidelity model evaluations versus 168 obtained with the conventional method.

1. Introduction

Robust Design is a design methodology for improving the quality of a product by minimizing the impact of uncertainties on the product performance. The objective of robust design is to optimize the mean performance while minimizing the variation of performance caused by various uncertainties. In the context of aerodynamic shape optimization, robust design implies that the performance (such as coefficient of drag, the lift-to-drag ratio, etc.) of the final configuration should be insensitive to the uncertainties in the operating conditions (e.g., free-stream Mach number) and the geometry (e.g., manufacturing uncertainties). An important component of robust design is Uncertainty Quantification (UQ), which may significantly increase the computational expense of the design process compared to the computational effort of deterministic optimization. This is particularly the case when high-fidelity analysis tools are involved in the design process in order to ensure sufficient accuracy. Therefore, it is important to develop and implement computationally efficient robust design methodologies while keeping the desired accuracy level in the optimization process.

Two types of input uncertainty should be considered in robust aerodynamic design studies: inherent (aleatory) uncertainty and epistemic uncertainty [1,2]. Aleatory uncertainty, which is probabilistic and irreducible, describes the inherent variation associated with the physical system (e.g., the operating conditions). Epistemic uncertainty [3] is reducible and described as lack of knowledge or information in any phase or operation of a design process (e.g., turbulence models used in CFD simulations). These two types of uncertainties usually co-exist (e.g., mixed uncertainties) in real-world

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http://dx.doi.org/10.1016/j.ast.2015.04.011
1270-9638/

Nomenclature

n	number of design variables	C_l	coefficient of lift
N	number of random variables	C_d	coefficient of drag
p_d	deterministic state variable vector	M	Mach number
SR	support region of random input variable	α	angle of attack in degrees
n_p	oversampling ratio	β	geometry parameter in thickness distribution formula for NACA 4-digit airfoils
p	order of polynomial chaos	Re	Reynolds number
$\boldsymbol{\xi}$	standard input random variable vector	N_h	number of high-fidelity CFD simulations
$p(\boldsymbol{\xi})$	probability density function of $\vec{\xi}$	N_f	number of low-fidelity CFD simulations
ψ	random basis function	N_{cost}	total design cost
a	coefficient in polynomial chaos expansion	LF	low-fidelity
a^*	stochastic function	HF	high-fidelity
μ	mean	CLF	corrected low-fidelity
σ	standard deviation		
N_t	number of terms in a total-order expansion		

systems. In mathematical terms, aleatory uncertainties are characterized by probability density functions with sufficient information on the type of the distribution. In order to characterize epistemic uncertainty, probabilistic methods are not suitable due to insufficient information about the uncertainty. One possible approach to model the epistemic uncertainty is to characterize it with intervals. For mixed uncertainty quantification, formulations that combine probabilistic methods and interval approach are usually sought. The aerodynamic response (e.g., the drag coefficient) should be in the form of the combination of probability distribution due to the effect of aleatory input uncertainty and interval distribution which indicate the effect of epistemic uncertainty.

This paper attempts to further reduce the computational cost of the robust design procedure introduced in Zhang et al. [4] and builds upon the recent study by the authors [5], which focused on robust optimization under inherent uncertainties only. The proposed approach is based on replacing the computationally expensive High-Fidelity (HF) CFD model by its inexpensive representation referred to as the Corrected Low-Fidelity (CLF) model. The Low-Fidelity (LF) model is evaluated using the same CFD solver but with a coarser mesh and relaxed convergence criteria. The misalignment between LF and HF models is reduced by means of Output Space Mapping (OSM) [6–9]. The OSM technique has traditionally been used as an auxiliary response correction method in the context of design optimization, with the LF model being corrected at each iteration using the HF model data accumulated during the process. In the proposed approach, the correction can only be performed once, for the points used for constructing the stochastic surrogate model based on Non-Intrusive Polynomial Chaos (NIPC) technique. Moreover, the CLF model has to be aligned sufficiently well with the HF model in the entire design space to be considered in the construction of the surrogate model subsequently utilized in the optimization process. Such an alignment is obtained by using design-variable-dependent multiplicative OSM set up with sufficient number of HF training samples.

In the next section, different robustness measures and objective function formulation for robust design depending on the input uncertainty type are given. The UQ approach, which is the point-collocation NIPC based stochastic expansions is described in Section 3. Further, the multi-fidelity approach involving the construction of the CLF model based on the HF model using OSM strategy is explained in Section 4. To demonstrate the multi-fidelity robust optimization methodology under mixed uncertainties, a CFD example is presented in Section 5 with Mach number considered as aleatory uncertainty and β (geometry) parameter as the epistemic uncertainty. The NACA airfoil shape parameters and the angle of attack are treated as deterministic design variables. Section 6 concludes the paper with important interpretations of the results obtained.

2. Problem formulation for robust optimization

2.1. Deterministic design

In general, the goal of Aerodynamic Shape Optimization (ASO) is to find a shape such that one or more performance metrics are optimized for a given operating condition(s), while at the same time fulfilling a set of constraints. Mathematically, the ASO problem consists of determining values of design variables $\mathbf{x} \in R^n$, such that the objective function $J : R^n \rightarrow R$ is minimized,

$$min \quad J(\mathbf{x}, \mathbf{Q}), \tag{1}$$

subject to constraint equations,

$$\mathbf{g}(\mathbf{x}, \mathbf{Q}) \leq 0, \tag{2}$$

where $\mathbf{Q}$ denotes the vector of conservative flow variables, and $\mathbf{g} : R^n \rightarrow R^m$ is a vector function containing m constraints. The flow variables must satisfy the governing flow equations, $\mathbf{R}$,

$$\mathbf{R}(\mathbf{x}, \mathbf{Q}) = 0. \tag{3}$$

The functions J and $\mathbf{g}$ are assumed to be continuous and differentiable over the design space of interest.

The problem formulation (1)–(3) is general and can be applied to different design approaches. The one-point and one-objective approach is widely adopted, where the aerodynamic surface is optimized for one operating condition with a single merit function. The most common example for this type of optimization is the lift-constrained drag minimization problem. Here, the goal is to improve the aerodynamic efficiency while maintaining a required lift. The objective function is set as

$$J = C_d, \tag{4}$$

where C_d is the drag coefficient and the lift constraint is

$$g = C_l^* - C_l \leq 0, \tag{5}$$

where C_l is the lift coefficient obtained for design $\mathbf{x}$, and C_l^* is the required lift coefficient. Parameters of the operating condition include the Mach number, M_∞, the Reynolds number, Re, and the angle of attack, α (which can be set as a design variable or it can be considered a state variable that is adjusted during the flow solution to satisfy (3)). Formally, we can say that the lift and drag coefficients are a function of the design variables, $\mathbf{x}$, and the state variables, $\mathbf{p} = [M_\infty \; Re \; \alpha]^T$, i.e., $C_d = C_d(\mathbf{x}, \mathbf{p})$ and $C_l = C_l(\mathbf{x}, \mathbf{p})$.