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Maximum endurance for battery-powered rotary-wing aircraft

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ABSTRACT

The paper presents an analytical framework for addressing the hovering time prediction of rotary-wing aircraft, with a particular focus on multi-rotor platforms. By imposing the balance between required and available power, the endurance expression is derived as a function of airframe features, rotor parameters, and battery capacity. The best endurance condition is also obtained in terms of optimum capacity and hovering time, by means of two approximate closed-form solutions. The proposed methodology was validated by means of numerical simulation and flight testing. Results show the effectiveness of the proposed approach.

1. Introduction

The interest in electrically powered Remotely Piloted Aerial Systems (RPAS) among industry and academia has shown a steep increase along the last decade. This is mostly due to the potentiality of such platforms to play a leading role within a wide range of applications in the field of aerial photography and 3D reconstruction [1], search and rescue [2] and risk management, natural landscape monitoring, and disaster prevention [3].

In particular, among the available fixed and rotary-wing configurations, multi-rotor platforms gained a particular relevance, thanks to the simple configuration, simplicity of use (also in confined spaces), and hovering and vertical take-off and landing capability, which all make them an interesting alternative to fixed-wing aircraft in many practical applicative scenarios. Also, with respect to a conventional helicopter, a multi-rotor shows an increased manoeuvrability and a faster response to external disturbances together with a more compact size. This is achieved by spreading the total disc area into multiple rotor units, [4,5]. Thus, to provide the required thrust, smaller propellers rotating at higher speed are employed, at the cost of a loss in efficiency with respect to the conventional, single rotor configuration. This, in addition to the limited duration/weight ratio typical of electrically driven systems, makes the endurance in the hovering condition a critical, but challenging, issue in the framework of the multi-rotor platform design process.

Studies addressing electric aircraft performance became available only in the last few years. Recent works present results related to fixed-wing aircraft, where the effects of Peukert's law on the available battery capacity are taken into account. In Ref. [6] this approach was first applied to flight vehicles, later on in Ref. [7] the best range condition is also derived, whereas in Ref. [8] endurance estimates are validated by means of an experimental investigation. With respect to multi-rotor platforms, Ref. [9] presents a novel configuration where endurance is increased by using higher diameter propellers for lift generation and smaller propellers for attitude control, whereas in Ref. [10] the authors present an application of a statistically-based sizing methodology to multi-rotor RPAS, allowing for the estimation of the gross take-off weight as a function of hovering time and payload requirements, through the analysis of available data from platforms currently on the market.

This paper presents the analytical study of battery-powered rotary-wing aircraft endurance in hovering flight condition, and its experimental validation carried out by using a multi-rotor platform. Starting from the balance equation of required and available power, and by taking into account airframe features, rotor figure of merit, and payload required power, two main results are obtained and discussed: 1) an analytical model for accurate estimate of hovering time as a function of on-board battery capacity, and 2) a set of approximate solutions for deriving the best endurance condition in terms of optimum capacity and maximum achievable hovering time for the cases when the figure of merit can be assumed to be constant, and the power required by the payload is small if compared to the hovering power. As a further contribu-

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tion, a simplified model not depending on rotor figure of merit is proposed, which can be used for estimating take-off weight during multi-rotor sizing at a preliminary/conceptual stage, [11].

The rest of the paper is structured as follows. In the next section the problem of hovering flight time is addressed and approximate analytical models describing the best endurance condition are derived. In the Results section numerical simulations and flight testing campaign data, validating the effectiveness of the proposed approach, are presented. A section of concluding remarks ends the paper.

2. Problem statement and solution

Let us consider a rotary-wing platform equipped with a payload for a specific mission. The total take-off weight can be expressed as

$$W_{to} = W_{eo} + W_p + W_b \tag{1}$$

where W_{eo} is the empty-operative weight that includes a) the frame weight (structure and rigging), b) the driving system weight (motors, regulators, and propellers), and c) the avionics weight (autopilot and communication system), W_p is the payload weight, and W_b is the battery weight.

For the purposes of the present analysis, the total power required for flight is

$$P_r = P_h + P_{ap} \tag{2}$$

where P_h is the power required for the hovering condition and P_{ap} is the power required for avionics and payload. For rotary-wing aircraft, the power required in hovering condition is given by [12]

$$P_h = \frac{W_{to}^{3/2}}{f\sqrt{2\rho A_t}} \tag{3}$$

where f is the figure of merit of the rotor, ρ is air density, and A_t is the total disc area. After naming $\lambda = \sqrt{2\rho A_t}$, by imposing the balance between the required and available power from the battery, the current draw, i, for the hovering condition is given by

$$i = \frac{P_r}{\mathcal{V}} = \frac{1}{\mathcal{V}}\left(\frac{W_{to}^{3/2}}{\lambda f} + P_{ap}\right) \tag{4}$$

where $\mathcal{V}$ is the battery voltage that, in general, is a function of both current draw and capacity. From the definition of the discharge ratio, $dC/dt = i$, it is straightforward to obtain the specific endurance

$$\frac{dt}{dC} = \frac{\mathcal{V}\lambda f}{W_{to}^{3/2} + P_{ap}\lambda f} \tag{5}$$

For a nominal battery capacity C_0, the actual available capacity C at the discharge rate i is provided by the Peukert equation, [13],

$$C = C_0\left(\frac{C_0}{it_0}\right)^{k-1} \tag{6}$$

where t_0 is the rated discharge time, and k is Peukert's coefficient which can be estimated by using the method described in Ref. [6]. By considering the complete discharge of the available capacity, flight endurance is given by the following integral:

$$t = \int_0^C \frac{\mathcal{V}\lambda f}{W_{to}^{3/2} + P_{ap}\lambda f}\, dC \tag{7}$$

Assumption 1. On the basis of validation data presented in Ref. [8], the battery voltage is supposed to be constant during the discharge.

For constant power applications, Li-Po cells show a linearly decreasing voltage as a function of residual capacity (with a negligible dependency on current draw) when discharged from the fully-charged voltage, $\mathcal{V}_f$, to the standard voltage, $\mathcal{V}_0$, [14]. Let η express the fraction of the nominal capacity where the discharge process shows a linear behaviour. The available capacity becomes

$$C = \eta C_0\left(\frac{\eta C_0}{it_0}\right)^{k-1} \tag{8}$$

By imposing in Eq. (7) the discharge at the equivalent constant voltage, $\mathcal{V}_e = (\mathcal{V}_f + \mathcal{V}_0)/2$, the endurance turns to

$$t = \frac{\mathcal{V}_e\lambda f}{W_{to}^{3/2} + P_{ap}\lambda f}\int_0^C dC = \frac{\mathcal{V}_e\lambda f}{W_{to}^{3/2} + P_{ap}\lambda f}\eta C_0\left(\eta\frac{C_0}{it_0}\right)^{k-1} \tag{9}$$

Assumption 2. Rotor figure of merit is supposed to be a slowly-varying power function of the rotor thrust. The following model is proposed:

$$f = f_0\left(\frac{W_{to}}{T_0 n}\right)^m \tag{10}$$

where T_0 is the rotor thrust at a reference percentage of throttle position (i.e. at 60%), and n is the number of rotors. Model parameters, f_0 and m, are obtained by available data from the manufacturer and/or by means of an experimental characterisation.

2.1. Maximum endurance condition

Letting α indicate the battery weight/energy ratio, aircraft take-off weight can be expressed as

$$W_{to} = W_0 + W_b = W_0 + \alpha\mathcal{V}_e C_0 \tag{11}$$

where $W_0 = W_{eo} + W_p$ is the zero-capacity weight, representing the weight of the aircraft without the battery system. By substituting Eqs. (4) and (10) into Eq. (7), and by taking into account Eq. (11), endurance in hovering flight is given by

$$t = t_0\left[\frac{\mathcal{V}_e\eta f_0\lambda}{t_0(T_0 n)^m}\right]^k \Phi(C_0) \tag{12}$$

where the function

$$\Phi(C_0) = C_0^k\left[(W_0 + C_0\mathcal{V}_e\alpha)^{m-3/2} + \frac{f_0\lambda}{(T_0 n)^m}P_{ap}\right]^{-k} \tag{13}$$

depends on the nominal battery capacity.

Provided that all the parameters in the bracketed coefficient of Eq. (12) are constant, the best endurance condition is obtained by taking the derivative of Φ with respect to C_0 and imposing $d\Phi(C_0)/dC_0 = 0$. The nominal capacity allowing for the maximum endurance is thus obtained by solving the equation

$$\frac{\mathcal{V}_e\alpha(2m-3)(W_0 + C_0\mathcal{V}_e\alpha)^{1/2}(T_0 n)^m}{(W_0 + C_0\mathcal{V}_e\alpha)^{3/2}(T_0 n)^m + P_{ap} f_0\lambda(W_0 + C_0\mathcal{V}_e\alpha)^m} + \frac{2}{C_0} = 0 \tag{14}$$

To the best of the authors' knowledge, an analytical solution to the problem cannot be found when $\Phi(C_0)$ assumes the form expressed in Eq. (13). Nonetheless approximate closed-form solutions can be obtained on the basis of two different simplifying assumptions.

Case 1. The rotor figure of merit is assumed to be constant, $f \approx f_0$. This assumption holds when f shows negligible variations within