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Analytical solution for the free vibration characteristics of the rotating composite beams with a delamination

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ABSTRACT

In this paper, the free vibration analysis of rotating delaminated composite beams with general lay-ups is analytically investigated. The Hamilton principle is used to derive the coupled governing differential equations and boundary conditions for a rotating delaminated beam taking into account the effects of shear deformation, rotary inertia, material couplings (bending–tension, bending–twist, and tension–twist couplings), and Poisson's effect. Both the free mode and constrained mode assumptions in the study of vibration of delaminated composites are used in the present analysis. Analytical solution for the natural frequencies and mode shapes is presented by incorporating the constraint conditions using the method of Lagrange multipliers. The accuracy of the results is verified from the convergence study of the natural frequencies and from the comparisons made with published results. The effects of various parameters such as delamination parameters, fiber angles, hub radius, material anisotropy and rotating speeds of the beam are studied in detail. Also, a comparison between the results of the free mode and constrained mode assumptions is conducted.

1. Introduction

The rotating beams made by composite materials are widely used in various engineering fields such as robotic manipulators, wind turbines and helicopter blades. This is due to their superior characteristics such as high strength-stiffness, lightweight, fatigue resistance etc. Contrary to these mechanical merits, they are subjected to a wide range of damages that can be induced during their fabrication or service life and significantly reduce their structural performance. One of the commonly encountered types of defects or damages in the multi-layered composite structures is delamination.

The composite material for a specific application usually requires the use of angle ply and unsymmetrical laminates. Thus, bending–tension, bending–twist, and tension–twist couplings will appear [1–5]. Furthermore, the Poisson effect, which is often neglected in one-dimensional laminated beam analysis, has very significant influence in the analysis of the angle-ply and unsymmetrical lay-ups of the beam [1–5].

Free vibrations analysis of the intact rotating beams and delaminated non-rotating beams based on the CBT and shear deformation theory has received a good amount of attention in the literature [6–27], whereas the dynamic analysis of the rotating delaminated beam has received attention only recently [28].

The free vibrations of an isotropic beam with a through-width delamination was studied by Wang et al. [6] by using four Euler–Bernoulli sub-beams connected at the delamination boundaries. According to this study dramatic interpenetration of the delaminated sub-beams was seen which was physically impossible to occur in the case of off-mid-plane delaminations. This is because the delaminated regions were assumed to deform "freely" without touching each other (known as the *free mode*) and thus have different transverse deformation. In other words, the mode shapes of the two segments of the delamination region are different and that the transverse displacement of the upper segment is greater than that of lower segment at all corresponding axial coordinates. Such a free displacement mode would give rise, in one half of the cycle of motion, to an overlap between the two segments, that is a downward displacement of the upper segment greater than that of the lower segment, or vice versa. To avoid this kind of incompatibility, Mujumdar and Suryanarayan [7] proposed a model in which they assumed that the delaminated layers are constrained to have identical transverse deformations. This model was called the *constrained mode* model in contrast with the free mode model [6]. However, the constrained mode model failed to predict the opening in the mode shapes found in the experiments by Shen and Grady [8]. Analytical and numerical solutions for beams with single and multiple delaminations have been presented by many

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Nomenclature

L	Beam length
$b \times h$	Rectangular cross-section of the beam
L_1	Delamination lengthwise location
L_2	Delamination length
r_0	Offset distance
hi	Thickness of the ith sub-beam
Ω	Rotating speed
ω_s	Natural frequency of the non-rotating intact beam
T	Kinetic energy
U	Potential energy
(A, I)	Area and second moment of area, respectively
R	Radius of gyration
ρ_i	ith layer density
(u, w)	The LCB mid-plane displacements in the $\hat{x}$ and $\hat{y}$ directions, respectively
$(\psi_{\hat{x}}, \psi_{\hat{y}})$	The mid-plane bending slopes
$N_{\hat{x}}$	In-plane force
$(M_{\hat{x}}, M_{\hat{x}\hat{y}})$	Bending and twisting moments
$Q_{\hat{x}\hat{z}}$	Resultant shear force
$\varepsilon_{\hat{x}}^0$	Mid-plane strain
$(\kappa_{\hat{x}}, \kappa_{\hat{x}\hat{y}})$	Bending and twisting curvatures
$\varepsilon_{\hat{x}\hat{z}}$	Shear strain
$\bar{A}_{ij}$	Modified extensional stiffness
$\bar{B}_{ij}$	Modified bending–extension coupling stiffness
$\bar{D}_{ij}$	Modified bending stiffness
A_{ij}	Transverse shear stiffness
F_i	Centrifugal force
ω	Circular frequency
k_s	Shear correction factor
E	Young modulus
(e_2, e_3)	Distances between the neutral axis of the sub-beams 2 and 3 with the neutral axis of the intact part, respectively.
$(\eta' = \Omega L^2\sqrt{\rho A/EI},\ \eta = \Omega L^2\sqrt{\rho/(E_{11}h^2)})$	Dimensionless rotating speed
$(\bar{\omega}' = \omega L^2\sqrt{\rho A/EI}, \bar{\omega} = \omega L^2\sqrt{\rho/E_{11}h^2})$	Dimensionless natural frequency
$(r = R/L, \alpha = r_0/L, \bar{L}_2 = L_2/L, \mu = M/\rho bhL, \sigma = J/\rho bhL^3)$	Non-dimensional parameters used in the numerical analysis

Abbreviation

LCB:	Laminated Composite Beam
CBT:	Classical Beam Theory
WPE:	With Poisson's Effect
WOPE:	Without Poisson's Effect

researchers based on the CBT [9–14] and shear deformation theories [15–17]. These studies emphasized on the influence of the free and constrained modes between the delaminated layers.

The free vibration characteristics of rotating intact beam have received considerable attention by the researchers [18–27]. Yokoyama [18] has presented the free vibrations analysis of a rotating Timoshenko beam by means of a finite element method. The beam has been divided into a number of two nodded elements with two degrees of freedom per each node namely transverse deflection and bending rotation. The mass and stiffness matrixes have been derived and the natural frequencies and corresponding mode shapes have been obtained using the eigen-value technique. Kou et al. [19] have investigated the influence of taper ratio, elastic root restraints, tip mass and rotating speed on the vibration of rotating non-uniform beams based on the CBT. Natural frequencies of rotating tapered Timoshenko beam has been presented for different combinations of the fixed, hinged and free end conditions by means of a new tapered finite beam element that accounts for the effect of shear deformation, rotary inertia, and the centrifugal stiffening due to beam rotation [20].

Chandra and Chopra [21] have investigated the free vibration analysis of coupled composite I-beams under rotation. The effects of shear deformation and rotary inertia have been considered in the analysis and the Galerkin method has been used to obtain the solution. Lee and Kuo [22] have derived the upper bound of the fundamental frequency of a rotating Timoshenko beam with elastically restrained boundaries by using the Rayleigh principle. Du et al. [23] have presented a convergent power series expression to solve analytically for the exact natural frequencies and modal shapes of rotating Timoshenko beams. The decoupled governing differential equation for transverse deflection has been obtained. Then the solution has been expressed in terms of axial coordinates. By substituting the solution into the governing differential equation and equating the coefficient of like powers of axial coordinates, the recurrence formula has been obtained for the power series coefficients. The effects of angular velocity, shear deformation and rotary inertia on the dynamic behavior of rotating beams have been evaluated.

Lin [24] has studied the dynamic analysis of a rotating Timoshenko beam with an elastically restrained root by transforming the partial governing differential equations into ordinary differential equations using the Laplace transform. Kaya [25] has applied the differential transform method to obtain the natural frequencies of a uniform Timoshenko beam with constant rotating speed. The free vibration analysis of rotating tapered beams has been investigated using the dynamic stiffness method by Su et al. [26]. The range of considered problems included beams for which the depth and/or width of the cross-section vary linearly along the length. Das et al. [27] have presented analytical solution for the free vibration of a rotating beam with nonlinear spring–mass system. The solution has been obtained by applying the method of multiple time scale directly to the nonlinear partial differential equations and the boundary conditions.

Analytical solutions have been developed by Liu and Shu [28] to study the free vibrations of delaminated rotating isotropic beam. The Timoshenko beam theory and both the free mode and the constrained mode assumptions in delaminated region have been used.

To the best of the author's knowledge, to date there are no analytical and numerical investigations in the literature on the free vibration analysis of the rotating generally LCB with an arbitrary delamination in which the Poisson effect, material couplings, rotary inertia and shear deformation have been considered. Therefore, the main objective of this paper is to try such kind of analysis. To do this, (a) by considering Poisson's effect, the material couplings (bending–tension, bending–twist, and tension–twist couplings), rotary inertia and shear deformation, the kinetic and potential energies expression of an LCB are derived, (b) by applying the Hamilton principle, the governing differential equations along with the boundary and compatibility conditions are obtained, (c) by choosing the Legendre polynomials and using its properties, the kinetic and potential energies, the constraints and then functional are made, (d) by extremizing the functional, a system of linear algebraic equations, which contain the Legendre polynomial coefficients and Lagrange multipliers are obtained, (e) next, substituting these linear algebraic equation in the constraints results in a system of homogeneous linear algebraic equations with