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Nonlinear analysis of functionally graded nanocomposite rotating thick disks with variable thickness reinforced with carbon nanotubes

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A B S T R A C T

In this paper the nonlinear elasticity solution of functionally graded nanocomposite rotating thick disks with variable thickness reinforced with single-walled carbon nanotubes (SWCNTs) is presented. Four distribution types of uniaxial aligned SWCNTs are considered: uniform and three types of functionally graded (FG) distributions along radial direction of the disk. The effective material properties of the nanocomposite disk are estimated by a micro-mechanical model. The governing nonlinear equations are based on the axisymmetric theory of elasticity with the geometric nonlinearity in axisymmetric complete form. The nonlinear graded finite element method (NGFEM) based on Rayleigh–Ritz energy formulation with the Picard iterative scheme is employed to solve the nonlinear equations. The solution is considered for four different thickness profiles, namely constant, linear, concave and convex. The effects of different types of distributions and volume fractions of CNTs and various types of thickness profiles on the displacement and stresses of the rotating disks as well as comparison between linear and nonlinear responses are investigated. The achieved results show that the displacement and stress fields can be controlled by changing the type of distribution and volume fraction of CNTs as well as the thickness profile. Moreover, the difference between linear and nonlinear results are noticeable in high angular velocities; thus, for obtaining accurate results, the geometric nonlinearity must be considered.

1. Introduction

Rotating disks are of practical concern in many fields of engineering, such as marine, mechanical and aerospace industry including gas turbines, gears, turbo-machinery, flywheel systems and centrifugal compressors. The stresses due to centrifugal load can have important effects on their strength and safety. Thus, control and optimization of stress and displacement fields can help to reduce the overall payload in aerospace industry.

In recent years, nano-structured materials such as nanocomposites have generated considerable interest in the material research community and became an attractive new subject in material science due to their potentially impressive mechanical properties. Carbon nanotubes (CNTs) have illustrated remarkable mechanical, thermal and electrical properties. For instance, they could potentially have a Young's modulus as high as 1 TPa and a tensile strength approaching 100 GPa [20]. These enormous advantages make them highly desirable candidates for the reinforcement of the polymer composites, provided that good interfacial bonding between CNTs and polymer and proper dispersion of the individual CNTs in the polymeric matrix can be assured [9].

The majority of researches performed on carbon nanotube reinforced composites (CNTRCs) are focused on their material properties [8,29,6,13,17]. Han and Elliott [11] by use of molecular dynamic simulation (MD) obtained the elastic modulus of composite structures reinforced with CNTs and studied the effect of volume fraction of SWCNTs on mechanical properties of nanocomposites. Hu et al. [12] by analyzing the elastic deformation of a representative volume element (RVE) under various loading conditions evaluated the macroscopic elastic properties of CNTRCs. Zhu et al. [36] studied the effect of CNTs on the mechanical properties of polymeric composites. Their results show that adding CNTs can greatly improve the Young's modulus. Due to dependency of the interaction at the polymer and nanotube interface on the local molecular structure and bonding, Odegard et al. [21], by utilizing an equivalent-continuum modeling method, proposed a constitutive model for CNTRCs.

Functionally graded materials (FGMs) are special composite materials, microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. This idea which was used for the first time by Japanese researchers [15], leads to the concept of FGMs. A wide range of

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researches have been carried out on FGMs in various fields of mechanics. Motivated by the concept of FGMs, Shen [26] presented a type of CNTRCs that the volume fraction of CNTs are graded with certain rules along desired directions and demonstrated that the use of FG-CNTRCs improves the mechanical properties of the structures. Zhu et al. [35] studied the bending and free vibration analyses of composite plates reinforced with SWCNTs using the finite element method based on the first order shear deformation plate theory. The effective material properties of the FG-CNTRC are graded in the thickness direction and are estimated according to the rule of mixtures. The three-dimensional free vibration of FC-CNTRC panels has been investigated by Yas et al. [30]. The boundary conditions are assumed to be simply support and the equations are solved by a generalized differential quadrature (GDQ) method. Shen [27] presented a postbuckling analysis for nanocomposite cylindrical shells reinforced with single-walled carbon nanotubes (SWCNTs) subjected to lateral or hydrostatic pressure in thermal environments. Sobhani Aragh and Yas [28] investigated the static and free vibration characteristics of continuously graded fiber-reinforced (CGFR) cylindrical shells based on three-dimensional theory of elasticity. The boundary conditions are assumed to be simply support and the equations are solved by a GDQ method. Moradi-Dastjerdi et al. [18] studied the free vibration of FG-CNTRC cylinders with a mesh-free method. The conditions are assumed to be axisymmetric where the effect of the waviness of the CNTs and its parameters are studied.

Moreover, numerous researches have been performed on analysis of functionally graded rotating disks. Durodola and Attia [7] studied the deformation and stress fields in functionally graded rotating disks using direct numerical integration of the governing differential equations as well as finite element analysis. Zenkour [33] investigated an analytical solution for functionally graded annular rotating disks under the plane-stress assumption with exponentially variable material properties. He later extended this solution to a variable thickness rotating disk [34]. Nie and Batra [19] obtained the equation of functionally graded rotating disks with variable thickness by using an Airy stress function and solved both analytically and numerically using the differential quadrature method. Kordkheili and Naghdabadi [16] presented a semi-analytical thermoelasticity solution for hollow and solid rotating axisymmetric disks made of functionally graded materials based on the plane-stress assumption. Their solution is based on dividing the radial domain into sub-domains. Callioglu et al. [5] investigated the stress analysis of functionally graded rotating disks and demonstrated that they have the capability of higher angular rotations compared with the homogeneous isotropic ones. Asghari and Ghafoori [1] proposed a semi-analytical three-dimensional elasticity solution for the rotating functionally graded problem suitable for thick disks. Zafarmand and Hassani [31] presented the elasticity solutions of 2D-FG thick rotating annular and solid disks with variable thickness. Shariyat and Mohammadjani [25] studied the stress analysis of rotating thick 2D-FG annular plates with non-uniform loads and elastic foundation. A second order point collocation method with forward–backward schemes was adopted to solve the system of the governing and boundary conditions.

According to literature, the studies deal with linear analysis of rotating disks. It has to be noted that in high speed advanced modern technologies such as gas turbines, turbochargers, centrifugal devices or machine tools, the speeds of rotary parts may approach up to $\omega = 20\,000$ rad/s. Thus, the application of linear analysis to high angular velocities may cause remarkable errors. Therefore, in order to obtain accurate results, the nonlinear analysis should be applied. Very few nonlinear analysis have been presented so far for rotating disks [10,3], in which their governing nonlinear equations are based on plate theories with von Karman large displacement scheme that do not involve nonlinear terms of radial displacement.

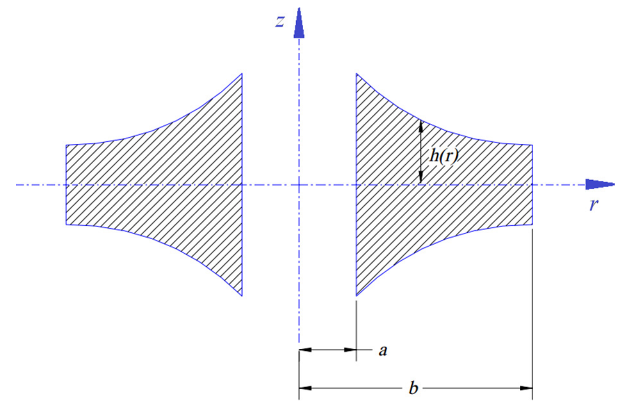

Fig. 1. Axisymmetric thick FG-CNTRC rotating disk.

To the best knowledge of the authors, there are no studies available in the literature on nonlinear analysis of FG-CNTRC rotating disks with variable thickness based on elasticity theory with the geometric nonlinearity in axisymmetric complete form. Thus, the purpose of this paper is to investigate the nonlinear analysis of thick FG-CNTRC rotating disks with variable thickness. Material properties are assumed to vary continuously along radial direction. The effective material properties of FG-CNTRC disk are estimated using a micro-mechanical model and the displacement and stress fields of FG-CNTRC disk for various types of distributions and volume fractions of CNTs as well as different types of thickness profiles are computed and compared. The difficulty in obtaining analytical solutions for the response of graded material systems comes from the dispersion of the heterogeneous material systems. Therefore, analytical or semi-analytical solutions are available only through a number of problems with simple boundary conditions. Besides, in the case of nonlinear analysis, the availability of such solutions becomes narrower. Thus, in order to find the nonlinear solution for a thick FG-CNTRC disk with variable thickness, powerful numerical methods such as nonlinear graded finite element method (NGFEM) are needed. The graded finite element incorporates the gradient of the material properties at the element scale in the framework of a generalized isoparametric formulation. Some studies can be found in the literature on modeling of nonhomogenous structures by using GFEM [14,2,32]. In these studies, it is shown that the conventional FEM formulation causes a discontinuous stress field whereas the graded elements give a continuous and smooth variation.

2. Problem formulation

In this section different types of CNTs' distributions along radial direction of the disk are investigated. The axisymmetric governing nonlinear equations of motion are obtained and nonlinear graded finite element method is employed for modeling the material nonhomogeneity and geometric nonlinearity.

2.1. Material properties in FG-CNTRC rotating disk

A thick FG-CNTRC disk of inner radius a, outer radius b and variable thickness $h(r)$ is considered. The geometry and coordinate system of the disk is shown in Fig. 1.

This FG-CNTRC disk consists of radially aligned SWCNTs and an isotropic matrix. Several studies have been published each with different focuses on mechanical properties of CNTRCs. However, for the sake of simplicity in the present study the rule of mixtures is employed and thus the effective material properties of CNTRC disk can be obtained as [26]: