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Nonlinear feedback control of self-sustained thermoacoustic oscillations

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ABSTRACT

Acoustic disturbances could undergo transient growth to trigger thermoacoustic instability in a combustion system with non-orthogonal eigenmodes, even with stable eigenvalues and in the presence of conventional linear controllers. In this work, nonlinear feedback control of thermoacoustic oscillations in a Rijke-type combustion system is considered. A thermoacoustic model with distributed monopole-like actuators is presented. To stabilize the thermoacoustic system, a nonlinear feedback controller is developed. It is shown that the feedback controller achieves both exponential decay of the flow disturbance energy and unity maximum transient growth. The performance of the controller is illustrated with two systems: one associated with one mode and one actuator, and the other with two modes and two actuators. The successful demonstration indicates that the developed nonlinear controller has the potential to be applied to combustion systems with multiple modes.

1. Introduction

Thermoacoustic instabilities are generated by a feedback interaction between combustion process and acoustic disturbances present in combustion systems [5,15,29]. When unsteady heat is added in phase with the pressure oscillations [2,19], acoustical energy increases. The pressure waves propagate within the combustor and partially reflect from boundaries to arrive back at the combustion zone. And they may cause more unsteady heat release. Under certain conditions, this feedback can result in large and damaging self-excited thermoacoustic oscillations [17] known as combustion instability. Such oscillations can cause structural damage, wearing at interfaces, flame flashback or blow-off, enhanced heat transfer and costly mission failure [15,27].

Thermoacoustic instability is currently a major challenge for land-based gas turbine and aero-engine manufacturers [4,10,11,13–15,18,25,26,28]. To stabilize combustion systems, passive and active control approaches have been used to break the coupling between the unsteady heat release and acoustic waves. Passive control [20] involves either applying acoustic dampers such as Helmholtz resonators [8,29] and acoustic liners [6,32] to increase the damping or redesigning the system, such as changing the flame anchoring position or changing the operating conditions. Passive approach is effective only over a limited range of operating conditions. Furthermore, it cannot respond to changes in operating conditions. Active control overcomes these difficulties.

Active control is generally based on either modulating the acoustic field by using a monopole-like sound source such as a loudspeaker [9] or modulating the unsteady heat release rate by using a secondary fuel injector [23,24]. The reader is referred to [5] and [17] for detailed reviews of thermoacoustic instability and feedback control techniques. The traditional linear controllers designed for a thermoacoustic system have focused on the dominant eigenmodes of the system. The objective of these controllers is to make the dominant eigenmodes decay exponentially. However, if the eigenmodes of the thermoacoustic system are non-orthogonal, energy exchange between eigenmodes can occur [1,30].

The non-orthogonality (i.e., non-normality) of thermoacoustic eigenmodes has received attentions recently [3,12,30] since much of the preceding work concludes that thermoacoustic system is non-normal. It has been shown that in a linearly stable system but with non-orthogonal eigenmodes, there can be significant transient energy growth of small perturbations before their eventual decay. If the acoustics transient growth is large enough, thermoacoustic instability might be triggered. In other words, small amplitude disturbances grow to amplitudes high enough to make nonlinear effects significant.

In this work, a nonlinear model of a simplified Rijke-type thermoacoustic system with multiple distributed actuators is first introduced. Then, a nonlinear feedback controller is designed to

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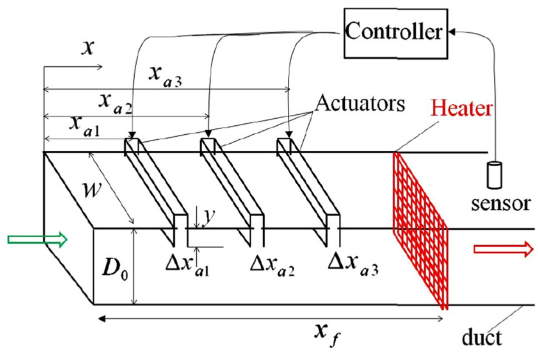

Fig. 1. A combustion system with distributed actuators modelled as monopole-like moving pistons.

exponentially stabilize the thermoacoustic oscillations. The effectiveness of the controller is demonstrated through a number of simulations. A preliminary version of this paper was presented as a conference paper [21].

2. Description of actuated thermoacoustic model

The thermoacoustic system examined in this paper, a horizontal Rijke tube with distributed actuators, is identical to that studied in [22,31]. However, for completeness, a brief description of the actuated thermoacoustic model is reproduced. In addition, the definition of the transient acoustical energy growth is introduced as a measure of the controller performance.

Consider a combustion system with distributed actuators modelled as monopole-like moving pistons (as shown in Fig. 1). Assume that $K \geq 1$ actuators are available for control purposes. Using tilde to denote the dimensional quantities and subscript 0 the mean values, the following non-dimensional variables can be defined:

$$u = \frac{\tilde{u}}{u_0}, \qquad p = \frac{\tilde{p}}{\gamma M_0 p_0}, \qquad \dot{Q}_s = \frac{\dot{\tilde{Q}}_s}{\gamma p_0 u_0},$$
$$x = \frac{\tilde{x}}{L_0}, \qquad t = \frac{\tilde{t} c_0}{L_0}, \qquad \frac{\delta(x - x_f)}{L_0} = \tilde{\delta}(\tilde{x} - \tilde{x}_f), \tag{1}$$

where x denote the location along the duct (actuators and heat source are located at x_{ak}, where $k = 1, \dots, K$, and x_f, respectively), t denotes time, p is the acoustic pressure, u is the velocity, M is the Mach number, c is the sound speed, L_0 is the length of the duct, and γ is the ratio of specific heats.

The non-dimensional acoustic equations of such a thermoacoustic system in the presence of K actuators can then be written as

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0, \tag{2}$$
$$\frac{\partial p}{\partial t} + \zeta p + \frac{\partial u}{\partial x} = (\gamma - 1)\dot{Q}_s \delta(x - x_f) + \gamma \sum_{k=1}^{K} \alpha_{ak} v_{ak} \delta(x - x_{ak}), \tag{3}$$

where ζp represents the losses due to thermo-viscous and friction damping, and α_{ak} describes the ratio of the cross-sectional area S_{ak} of the kth actuator to the cross-sectional area S of the duct, i.e., $\alpha_{ak} = S_{ak}/S$. The non-dimensional heat release rate $\dot{Q}_s$ can be written as [9]

$$\dot{Q}_s = \mathcal{K}\left[\sqrt{\left|\frac{1}{3} + u_f(t - \tau)\right|} - \sqrt{\frac{1}{3}}\right], \tag{4}$$

with

$$\mathcal{K} = \frac{2L_w(T_w - \bar{T}_0)}{\sqrt{3u_0} S \gamma p_0}\sqrt{\pi \lambda c_v \rho_0 \frac{d_w}{2}}, \tag{5}$$

where d_w, L_w, and T_w denote the diameter, length and temperature of the heated wire, respectively; ρ denotes density, T temperature, λ and c_v are the thermal conductivity and specific heat capacity at constant volume, respectively; and τ denotes the non-dimensional time delay describing the difference between the time when the oncoming velocity perturbation acts and the time when the corresponding heat release is felt. The values of these parameters for the thermoacoustic system example studied in this paper are given in Appendix A. The gas is assumed to be perfect, inviscid and non-conductive. The Mach number of the mean flow is assumed to be negligible and the heating element acoustically compact.

The acoustic pressure p and velocity u inside the duct can be written as a superposition of the duct natural modes as:

$$p(x,t) = -\sum_{j=1}^{N} \frac{\sin(j\pi x)}{j\pi} \dot{\eta}_j(t), \tag{6}$$

$$u(x,t) = \sum_{j=1}^{N} \cos(j\pi x) \eta_j(t), \tag{7}$$

where N represents the number of modes considered in the numerical discretization.

The actuation signal v_{ak} of the kth monopole-like source such as loudspeaker [7] can now be expressed as

$$v_{ak} = \mathcal{R}_k u(x_{ak}) + \mathcal{S}_k p(x_{ak}), \tag{8}$$

where $\mathcal{R}_k$ and $\mathcal{S}_k$ are dimensionless control parameters of the actuators. Discretizing the governing equations by using Eqs. (3), (6), (7), and simplifying yields

$$\frac{\ddot{\eta}_j}{j\pi} + j\pi \eta_j + \zeta_j \frac{\dot{\eta}_j}{j\pi} = -2(\gamma - 1)\dot{Q}_s(x_f, t - \tau)\sin(j\pi x_f) - 2\gamma \sum_{k=1}^{K} \alpha_{ak} v_a(x_{ak}, t)\sin(j\pi x_{ak}). \tag{9}$$

Note that in Eq. (9) the damping ζ is taken into account by assigning a damping parameter ζ_j to each mode. For the thermoacoustic system examined in this paper, p and $\frac{\partial u}{\partial x}$ are both set to zero at the ends of the duct (i.e., there is no acoustical energy being dissipated at the end boundaries). Furthermore, the acoustic waves are assumed to be planar such that there is also no acoustic energy being dissipated in the viscous and thermal boundary layers at the duct walls. Both types of dissipations are modelled by the damping parameter for each mode:

$$\zeta_j = c_1 j^2 + c_2 j^{0.5}, \tag{10}$$

where c_1 and c_2 are the same for each mode. This model, which is based on the correlations developed in [16], has been widely used [3,12].

To prevent the 'triggering' of initially low-amplitude perturbations that can lead to a nonlinear limit cycle, non-normality effect must be considered in the design of a controller. To characterize the transient growth of flow disturbances, it is necessary to define a measure in the system. We choose the total acoustical energy $E(t)$ per unit cross-sectional area as the measure consisting of a kinetic component and a potential component. The acoustical energy is defined in terms of dimensionless variables as