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Linear and nonlinear control strategies for formation and station keeping of spacecrafts within the context of the three body problem

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ABSTRACT

The problem of spacecraft formation control and reconfiguration for halo orbit around the second libration point (L_2) of the Sun–Earth Three Body (TB) system is investigated. Station keeping, reconfiguration and precision formation control of spacecrafts on halo orbits are performed via the use of the nonlinear Integral Sliding Mode (ISM) method as well as the optimal closed loop Linear Quadratic Regulator (LQR) approach. In this regard the nonlinear relative dynamics of deputy–chief spacecrafts are derived within the concept of the three body problem. The behavior of the two controllers are compared for different tasks of the formation mission in order to determine the more preferred strategy that fulfills the desired response behavior. In addition the controllers are evaluated for robustness in terms of their compensation against the major source of environmental disturbance, namely the Moon's gravity perturbation.

1. Introduction

Better understanding of the universe has been made possible through many space missions equipped with high precision instruments needed to observe the Earth and beyond more accurately. One such useful instrument for outer space observation is massive telescopes for interferometry purposes. As launch and placement of these kinds of huge telescopes is costly if not impossible, alternative innovative solutions in the form of spacecraft's formation have been proposed to this problem [3,24]. Distributed Space Systems (DSS) are one of the key approaches that can bring such ambitious missions into reality. Projects such as Darwin, Terrestrial Planet Finder (TPF), Micro-Arcsecond X-Ray Interferometry Mission (MAXIM) are among the many examples of DSS systems [10]. In these applications the role of spacecrafts formation is intuitively explicit. In fact many of these projects are defined for non-Keplerian orbits about the Lagrangian points of the Sun–Earth TB system. The Darwin project is also defined for halo orbits around the L_2 Libration point [3,24].

At the same time, one of the key requirements for DSS formation flying is precision control, that is of vital importance for interferometric missions needing accurate relative positioning [23]. Reconfiguration of formation is another important issue for DSS formation. In addition in many cases formation keeping and control are largely dependent on the leader spacecraft being stationed on the desired halo orbit, which by itself may be a formidable challenging task.

Farquhar [5] investigated halo orbits around the libration points of Three Body Problem (TBP). Gurfil and Kasdin [9] investigated the formation control of non-Keplerian orbit trajectory utilizing LQR control. Howell and Marchand of Purdue University have studied several projects on the TBP and formation flight in the vicinity of libration points. They have focused on formation keeping via continuous control, discrete control strategies for deployment and discrete formation keeping for unconventional configurations [12,17,18]. Gurfil et al. [10] worked on nonlinear adaptive neural control of formation flying, but have utilized an approximate model. Beichman and Gomez et al. [2] investigated mission design for the TPF project considering distributed spacecraft systems around the L_2 libration point. Gong et al. [8] have studied formation reconfiguration with impulse maneuvers via genetic algorithm to optimize the energy consumption. Pernicka et al. [22] researched on impulsive maneuvering to keep two satellites within a required tolerance. K. Shahid and K.D. Kumar [25] have also studied the spacecraft formation control at L_2 using solar radiation pressure. Peng et al. [20] utilized periodic optimal control to stabilize station and formation keeping of periodic orbits around the libration point with continuous low thrust. Similarly, Park and Choi [19] studied the problem of formation reconfiguration and station keeping of

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Nomenclature

F	the uncertainty bound of system
$\vec{f}^{moon}$	the Moon's gravitational perturbation vector
$\hat{f}$	estimation of dynamics of the system
H	final state gain positive semi-definite matrix
J	performance index of optimal control
K	LQR control gain matrix
k	ISM control discontinuity gain
$\bar{k}$	ISM smoothed control discontinuity gain
m_1	the Sun mass
m_2	the Earth mass
m_3	the Moon mass
m_4	the spacecraft mass
Q	LQR states gain positive semi-definite matrix
R	LQR control input gain positive definite matrix
$\vec{r}$	relative vector from the Sun to the Earth
$\vec{R}$	relative vector from the Earth to the Moon
r_1	distance between the Sun and spacecraft
r_2	distance between the Earth and spacecraft
r_{10}	distance between the Sun and leader spacecraft
r_{20}	distance between the Earth and leader spacecraft
s	weighted sum of position error, velocity error and integral of position error
$\vec{u}$	control input vector
$\hat{u}$	ISM best approximation of control
u, v, w	Components of the velocity vector of each spacecraft
X	the states of the system in ISM control
x, y, z	Components of the position vector of each spacecraft
$\tilde{x}$	difference between actual and desire trajectory in ISM control
$\vec{\rho}$	position vector of spacecraft with respect to the Sun–Earth barycenter
$\vec{\rho}_0$	position vector of leader spacecraft
μ	mass ratio of primaries
$\Delta\vec{u}$	relative control input vector
$\Delta\vec{f}^{moon}$	the Moon's gravitational relative perturbation vector
Δx	relative position of follower spacecraft along the x axis
Δy	relative position of follower spacecraft along the y axis
Δz	relative position of follower spacecraft along the z axis
$\Delta\vec{\rho}$	relative position vector from leader spacecraft to follower spacecraft
η	a positive constant denoting the time to reach the boundary layer
σ	total mass ratio
λ	negative of sliding surface slope
ϕ	boundary layer thickness

Earth orbiting satellites using LQR control and compared their result with those of linear parameter varying (LPV) controls. Wang et al. [28] developed an eigenstructure assignment control for formation keeping around the liberation point. Peng et al. [21] have further investigated a surrogate based parameter optimization approach for optimal rendezvous trajectory design in the Sun–Earth system. More recently, in 2014 Folta et al. [6] studied the problem of Halo orbit station keeping using optimal control strategy and also performed the stability analysis of the problem. Huang et al. [13] studied feasibility of using Coulomb forces for satellite formation control around the libration points orbit using indirect robust control.

The literature surveys show that the research in this area is live and ongoing, where still there are new ideas that can potentially contribute toward the betterment and efficiency of these missions. In this respect, the current study derives the linear and nonlinear relative dynamics of spacecrafts within the concept of the three body problem. Subsequently two types of controller are designed and analyzed for station keeping and formation control. Nonlinear Integral Sliding Mode (ISM) control as well as a time varying optimal closed loop Linear Quadratic Regulator (LQR) are developed and compared for Station Keeping (SK) with consideration of the Moon's perturbing effects. In addition the problem of continuous Precision Formation keeping and Reconfiguration (PFR) control is tackled for the deputies using the nonlinear sliding mode control. The controllers are analyzed in terms of performance, robustness and energy consumption for both SK as well as the PFR parts of the mission. Though, the general dynamics of the CRTBP is not new, application, comparison and analysis of the two utilized control methods implemented on all phases of the formation problem using the relative equations of motion consisting of station and formation keeping plus the reconfiguration are novel and yet not reported in the literature.

This paper is arranged as follows: Section 2 initially presents the dynamic model of the CRTBP, followed by derivation of the relative spacecrafts motion within the three body environment; moreover the corresponding linearized equations of motion are also derived in this section for utility in linear controller design. The two utilized control strategies are described, implemented and compared in Sections 3 and 4 for the station keeping phase of the formation problem. Similarly, the formation keeping and reconfiguration phases of control design are covered in Sections 5 and 6, using both the LQR and the ISM approaches. Finally, concluding remarks and future research prospects are addressed in Section 7.

2. Dynamical model

Circular Restricted Three Body Problem (CRTBP) as the best idealization of the TBP is investigated and utilized in many past and current works [7,15]. As this model neglects the effect of the spacecraft on the primaries known circular mutual motion, it is called the circular restricted TBP. Libration points are conspicuous in the fundamental plane of the rotating frame (see Fig. 1). As the dynamics of the system is extensively discussed in the literature, only the governing differential equations of CRTBP are presented in non-dimensional form [4,7,15]. It is also necessary to mention that in the non-dimensional problem: one year equals to 2π time unit (TU), while the physical distance between primaries is equivalent to one distance unit (DU).

$$
\begin{aligned}
\ddot{x} &= x + 2\dot{y} - \frac{(1-\mu)}{r_1^3}(\mu + x) - \frac{\mu}{r_2^3}\big(x - (1-\mu)\big) \\
\ddot{y} &= -2\dot{x} + y - \frac{(1-\mu)}{r_1^3}y - \frac{(\mu)}{r_2^3}y \\
\ddot{z} &= -\frac{(1-\mu)}{r_1^3}z - \frac{(\mu)}{r_2^3}z \\
r_1 &= \sqrt{(x+\mu)^2 + y^2 + z^2} \\
r_2 &= \sqrt{\big(x - (1-\mu)\big)^2 + y^2 + z^2}
\end{aligned} \tag{1}
$$

The reference halo orbit (around L_2) for formation keeping is found through shooting method [27], the Lagrangian point planned to be utilized also by the Darwin's project [3]. The period of this halo orbit is about 3.0930 TU which means 179.8 days. Due to numerical errors involved in the Halo trajectory and perturbations