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Simple finite-time attitude stabilization laws for rigid spacecraft with bounded inputs

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A B S T R A C T

The problem of finite-time attitude stabilization of a rigid spacecraft is investigated. Homogeneous system theory is utilized to design a simple nonlinear proportional-derivative-type (PD-type) saturated finite-time controller (SFTC), which can accommodate its form to different attitude parameterizations, such as quaternion, Rodrigues parameters (RP) and modified Rodrigues parameters (MRP). The proposed SFTC is inertia-independent and yields bounded control torques, in addition to finite-time convergence. Specially, the quaternion-based SFTC avoids the unwinding phenomenon associated with traditional continuous quaternion-based control laws and achieves, as shown by the Monte Carlo analysis, a faster convergence than a previous quaternion-based finite-time attitude stabilization law. Numerical examples are presented to verify the efficacy of the proposed method.

1. Introduction

The attitude control of a rigid body long has been a research interest [21], due to its potential applications to abundant mechanical systems such as robotics, aerial and underwater vehicles, and single or multiple spacecraft. Various control techniques have been applied to deal with the pointing control, large angle maneuver, and attitude tracking of a rigid spacecraft, etc., and extensive results have been gained [17].

The attitude of a rigid spacecraft actually evolves on the set of 3×3 rotation matrices SO(3), which is a compact manifold and is not equivalent to a linear vector space. This special topological feature prevents the existence of a continuous feedback law that globally asymptotically stabilizes a desired attitude on SO(3) [1]. In other words, continuous or smooth attitude control systems always possess multiple equilibria. SO(3) can be parameterized in numerous ways, among which unit quaternion is a redundant set of coordinates, double covering SO(3) [17]. The same topological obstruction, however, exists on the quaternion space. For the quaternion control laws in [22,23], Lyapunov stability of the desired physical attitude fails, resulting in the undesirable unwinding phenomenon. More precisely, although two quaternion equilibria both represent the same orientation, if the initial condition is close to the unstable equilibrium, the control law will drive the spacecraft to rotate almost a full revolution to the stable equilibrium. Such an unnecessary large angle slew will consume a considerable amount of control energy.

To overcome this problem, the set control approach was introduced in [10,11] to stabilize both two equilibria of the quaternion control system, yielding a discontinuous switching when initiating the control. Another discontinuous control law designed in [6], however, may induce the chattering problem in the vicinity of the switching surface due to sensor noise or disturbances. Another solution to eradicate unwinding is the geometric control approach on SO(3), which utilizes the uniqueness of rotation matrices in representing the attitude. In [16], a smooth attitude tracking law was constructed directly on SO(3). The resulting closed-loop systems are almost globally asymptotically stable, which means that the desired equilibrium is asymptotically stable except for a set of initial conditions with zero measure. Such kind of stability is weaker than global asymptotic stability but can be useful in practice, since a set of zero measure implies that the probability of entering such a set is zero.

Another problem is that most of the traditional attitude control laws are essentially smooth and can only yield asymptotic or, at best, exponential convergence of the closed-loop trajectories. In contrast, nonsmooth control laws may drive the system states to the equilibrium in finite time [2], which implies faster convergence

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and stronger disturbance rejection than asymptotic or exponential stabilization laws [10,11].

Recently, several investigations [5,10–12,24] have considered the application of the finite-time control method to spacecraft attitude control. The resulting control laws in [5,10,11], however, rely on an accurate system model to for implementation. Adaptive terminal sliding mode (TSM) and nonsingular terminal sliding mode (NTSM) control laws, which are robust to external disturbance and inertia uncertainties, were proposed in [12,24] to stabilize the spacecraft attitude with finite-time convergence. All the preceding finite-time control schemes, however, are relatively complicated in their forms and do not consider the input saturation problem. In other words, the actuators are assumed to provide any required control torque, whereas physical saturation constraints commonly exist among actuators. Ignoring this feature may severely degrade the performance of the control laws or even lead to instability of the system [18,20]. In [8], a velocity-free finite-time stabilization law was proposed. A switching type finite-time stabilization law was constructed in [4], yielding bounded control torques. The knowledge of the spacecraft inertia, however, is indispensible in order to effectively execute the switching logic, and thus this control law still depends on an accurate system model. Simple finite-time stabilization laws based on Rodrigues parameters (RP) and modified Rodrigues parameters (MRP) were also designed in [19], but the actuator saturation problem was considered. A constructive method was proposed in [15] to design finite-time attitude stabilization law on SO(3) and this method was then applied to the stabilization of simple mechanical systems [14]. The stabilization law in [15], however, is model-dependent and does not take the actuator saturation into account.

In this paper, the attitude stabilization problem of a rigid spacecraft is revisited. A simple nonlinear proportional-derivative-type (PD-type) saturated finite-time controller (SFTC) based on quaternion is designed to stabilize the spacecraft attitude. By means of the homogeneous system theory [7], almost global asymptotic stability and local finite-time stability of the two desired quaternion equilibria representing the same physical orientation are strictly proved. This property not only overcomes the unwinding phenomenon but also yields faster convergence and better disturbance rejection than asymptotically or exponentially stable systems. In this sense, the resulting almost global finite-time stability is stronger and more desirable than the almost global asymptotic stability, and can be viewed as the strongest stability for continuous attitude control systems, given the topological obstruction on attitude controls [1]. Apart from the finite-time stability, the SFTC achieves 1) robustness to inertia uncertainties, 2) the avoidance of the unwinding phenomenon associated with traditional continuous quaternion control laws, and 3) bounded torque input. In contrast, the linear or nonlinear PD regulators in [4,6,16,18,19,21–23] can only achieve part of the preceding merits.

In addition, the quaternion-based SFTC is extended to other attitude parameterizations, such as RP and MRP, yielding RP-based and MRP-based SFTCs in a similar structure. On the other hand, the quaternion-based SFTC in this paper can be viewed as a considerable improvement of the finite-time stabilization law in [4], by eliminating its switching logic and simplifying its structure significantly. A Monte Carlo analysis shows that such an improvement enables much faster convergence than the control law in [4], in addition to the inertia-free property.

The remainder of this paper is organized as follows. In Section 2, the equations of a rigid spacecraft using unit quaternion, RP and MRP are recalled, preceded by the general properties and stabilization results associated with homogeneous systems. In Section 3, the SFTCs based on quaternion, RP and MRP are derived and the stability of the resulting closed-loop systems is strictly proved. Illustrative simulation results are presented in Section 4 and conclusions are given in Section 5.

2. Preliminaries and system model

2.1. Mathematical preliminaries

Throughout this paper, let I_n denote the index set $\{1, 2, \cdots, n\}$ and $\|\cdot\|$ denote the Euclidean norm on $\mathbb{R}^n$. Given $\varepsilon > 0$ and a weight vector $\boldsymbol{r} = (r_1, \cdots, r_n)$ ($r_i > 0$, $\forall i \in I_n$), a dilation operator $\Delta_\varepsilon^{\boldsymbol{r}}$ is defined by $\Delta_\varepsilon^{\boldsymbol{r}}\boldsymbol{x} = [\varepsilon^{r_1}x_1, \cdots, \varepsilon^{r_n}x_n]^T$ for $\forall \boldsymbol{x} = [x_1, \cdots, x_n]^T \in \mathbb{R}^n$ [3]. In what follows, some basic definitions and properties on homogeneous systems, which will be utilized in proving the main results, are introduced.

Definition 1 *(Homogeneity).* (See [3].) Consider the system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$, $\boldsymbol{f}(0) = 0$, $\boldsymbol{x}(0) = \boldsymbol{x}_0$, $\boldsymbol{x} \in \mathbb{R}^n$, where $\boldsymbol{f} : U \mapsto \mathbb{R}^n$ is continuous on an open neighborhood U of the origin. Then, a continuous vector field $\boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}), \cdots, f_n(\boldsymbol{x})]^T \in \mathbb{R}^n$ is said to be homogeneous of degree $k \in \mathbb{R}$ with respect to a dilation $\Delta_\varepsilon^{\boldsymbol{r}}$ if $f_i(\Delta_\varepsilon^{\boldsymbol{r}}\boldsymbol{x}) = \varepsilon^{r_i+k} f_i(\boldsymbol{x})$ for $\forall i \in I_n$, $\forall \boldsymbol{x} \in \mathbb{R}^n$, and any $\varepsilon > 0$, where $k > -\min\{r_i, i \in I_n\}$. System $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ is said to be homogeneous if $\boldsymbol{f}(\boldsymbol{x})$ is homogeneous.

Lemma 1. *(See [7].) Consider the following system:*

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \hat{\boldsymbol{f}}(\boldsymbol{x}), \qquad \boldsymbol{f}(0) = 0, \quad \boldsymbol{x} \in \mathbb{R}^n \tag{1}$$

where $\boldsymbol{f}(\boldsymbol{x})$ is a continuous homogeneous vector field of degree $k < 0$ with respect to a dilation $\Delta_\varepsilon^{\boldsymbol{r}}$, and the perturbation vector field $\hat{\boldsymbol{f}}(\boldsymbol{x})$ satisfies $\hat{\boldsymbol{f}}(0) = 0$. Assume that $\boldsymbol{x} = 0$ is an asymptotically stable equilibrium of the system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$. Then, $\boldsymbol{x} = 0$ is a locally finite-time stable equilibrium of system (1) if

$$\lim_{\varepsilon \to 0} \frac{\hat{f}_i(\Delta_\varepsilon^{\boldsymbol{r}}\boldsymbol{x})}{\varepsilon^{r_i+k}} = 0, \quad i \in I_n, \ \forall \boldsymbol{x} \neq 0 \tag{2}$$

Moreover, if system (1) is (almost) globally asymptotically stable and locally finite-time stable, it is (almost) globally finite-time stable.

2.2. Equations of attitude motion

Denote S^2 as the 2-dimensional unit sphere. Let $\boldsymbol{\eta} \in S^2$ denote a unit vector along the Euler axis and ϕ denote the rotation angle around $\boldsymbol{\eta}$. Then, the unit quaternion $\boldsymbol{q} = [q_0, q_1, q_2, q_3]^T = [q_0, \boldsymbol{q}_v^T]^T$ can be defined as $q_0 = \cos(\phi/2)$ and $\boldsymbol{q}_v = \sin(\phi/2)\boldsymbol{\eta}$. Usually, q_0 and $\boldsymbol{q}_v$ are called the scalar and vector parts of the unit quaternion. It should be noted that the set of unit quaternion is a 3-dimensional unit sphere embedded in $\mathbb{R}^4$, i.e., $\boldsymbol{q} \in S^3$ and $q_0^2 + \boldsymbol{q}_v^T\boldsymbol{q}_v = 1$. The attitude kinematics in terms of unit quaternion are given by

$$\begin{bmatrix} \dot{q}_0 \\ \dot{\boldsymbol{q}}_v \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -\boldsymbol{q}_v^T \\ \boldsymbol{E}(\boldsymbol{q}_v) \end{bmatrix}\boldsymbol{\omega}, \quad \text{with } \boldsymbol{E}(\boldsymbol{q}_v) \triangleq \boldsymbol{q}_v^\times + q_0\boldsymbol{I}_3 \tag{3}$$

where $\boldsymbol{I}_3$ denotes the 3×3 identity matrix and $\boldsymbol{\omega} \in \mathbb{R}^3$ is the angular velocity with respect to the inertial frame. The operator $\boldsymbol{q}_v^\times$ denotes a skew-symmetric matrix generated by $\boldsymbol{q}_v$:

$$\boldsymbol{q}_v^\times = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

The RP can be defined by $\boldsymbol{\rho} = \tan(\phi/2)\boldsymbol{\eta}$, allowing $\pm 180°$ nonsingular rotations, whereas MRP can be defined by $\boldsymbol{\sigma} = \tan(\phi/4)\boldsymbol{\eta}$, allowing $\pm 360°$ nonsingular rotations. Note that shadow sets of MRP can be used to avoid its singularities [17]. The attitude kinematics in terms of RP and MRP are respectively given by