Aerospace Science and Technology 42 (2015) 249–258

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Robust multiple model adaptive estimation for spacecraft autonomous navigation

K. Xiong *, C.L. Wei, L.D. Liu

Science and Technology on Space Intelligent Control Laboratory, Beijing Institute of Control Engineering, Beijing 100190, China

ARTICLE INFO

Article history:
Received 13 March 2013
Received in revised form 17 December 2014
Accepted 22 January 2015
Available online 28 January 2015

Keywords:
Multiple model adaptive estimation
Robust Kalman filter
Spacecraft autonomous navigation
X-ray pulsar

ABSTRACT

This paper focuses on the development of a robust multiple model adaptive estimation (RMMAE) algorithm and its performance analysis. The main goal of this work is to enhance the robustness of the estimator against the model parameter identification error. A proof is provided that shows the convergence property of the proposed algorithm. Further analysis shows that the RMMAE algorithm guarantees a bounded energy gain from the model parameter identification error to the estimation error. The performance of the RMMAE is evaluated via simulations for spacecraft autonomous navigation. Simulation results demonstrate the effectiveness of the new algorithm compared with the extended Kalman filter (EKF), the unscented Kalman filter (UKF), the robust Kalman filter (RKF) and the multiple model adaptive estimation (MMAE).

1. Introduction

When system model and noise statistic are known, the Kalman filter (KF) and the extended Kalman filter (EKF) are usually implemented to obtain the optimal estimation of the state. However, in some practical applications, system models cannot be accurately acquired. In the presence of model uncertainty, the filtering performance may be degraded. This has motivated many studies of multiple-model adaptive estimation (MMAE) [2,15,24,27,29]. The MMAE algorithm is an effective approach to handle problems with model uncertainty. In the multiple-model approach, the uncertainty is approximated by a set of models. The MMAE uses a parallel bank of KFs (or EKFs) to provide multiple estimates, where each filter corresponds to a model in the predetermined model set. The final state and covariance estimate is provided by the weighted sum of each filter's estimate. Under certain conditions, the weight associated with the correct model will converge to 1, and the other weight will converge to 0. In this way, the MMAE is able to choose the appropriate model adaptively.

During the past four decades, the MMAE algorithms have been successfully implemented in various applications, such as target tracking [17], fault diagnosis [12] and bias calibration [34]. Three generations of MMAE have been characterized. The first is the classical algorithm presented in [21]. The second is the interacting multiple-model (IMM) algorithm [4] and the third is variable structure multiple-model (VSMM) algorithm [16]. The classical algorithm assumes that the model does not vary with time. In the IMM algorithm, the jumps in system model are taken into consideration. The character of the VSMM algorithm is that the model set is assumed variable and made adaptive based on measurements.

In the MMAE, an uncertain model parameter is assumed to belong to a finite set. In actual engineering problems, the uncertain model parameter is usually known to be in a bounded region. For the implementation of the MMAE, a discrete model set is constructed by selecting a number of points in the bounded region. An approximation is introduced thereby. The most likely parameter in the discrete model set may be not exactly equal to the true model parameter. In this paper, the difference between the most likely parameter and the true parameter is called as the parameter identification error. The magnitude of the parameter identification error depends on the discretization level of the model set. It may be rather large due to a coarse discretization. In this case, the filtering performance would be degraded. Generally, the greater is the number of models, or the denser is the covering of the bounded region by discrete points, the more accurate the approximation will be. To achieve a fine discretization level, a large number of models are required. However, the use of more models increases the computational burden considerably. When the model set is extraordinarily large, the MMAE will be computationally infeasible.

Although the MMAE is designed to deal with the model uncertainty, some parameter identification error remains when a discrete model set is adopted to approximate the uncertain model parameter. The magnitude of the parameter identification error is related to the density of the discrete elements in the model set.

* Corresponding author.
E-mail address: tobelove2001@vip.tom.com (K. Xiong).

http://dx.doi.org/10.1016/j.ast.2015.01.021

In this paper, the parameter identification error is seen as the remained uncertainty, and the robust filtering technique is adopted to cope with this uncertainty. The robust filtering problem has been extensively studied by academic researchers. Many robust or H_∞ filtering approaches have been developed. See [6,9,14,30,32,35] and the references therein. Generally speaking, the robust filtering approach guarantees an upper bound to the estimation error covariance despite the parameter uncertainties, and subsequently minimizing this upper bound, while the H_∞ filtering theory aims at designing an estimator that ensures a bound on the energy gain from the uncertainties to the estimation error. The Riccati equation and linear matrix inequality approaches are frequently exploited in designing robust filters.

This paper proposes a robust multiple model adaptive estimation (RMMAE) algorithm. In order to suppress the effect of the parameter identification error, the state estimation for each model is implemented by the robust Kalman filter (RKF) [33]. The computational burden of the RKF is roughly the same as that of the EKF as its equation resembles that of the EKF. Theoretical proofs are provided that show the convergence and robustness properties of the novel algorithm.

The rest of the paper is organized as follows. Section 2 formulates the system model and the RMMAE algorithm. Section 3 provides the proof of convergence. The robustness analysis of the algorithm is presented in Section 4. Section 5 illustrates the application of the algorithm to the spacecraft autonomous navigation based on X-ray pulsars. Section 6 provides the simulation results including comparisons on the EKF, the UKF, the RKF and the MMAE. Finally, our conclusion is drawn in Section 7.

2. RMMAE algorithm

The RMMAE algorithm for nonlinear systems with parameter uncertainty is formulated, which is a combination of the MMAE algorithm shown in [10] and the RKF algorithm shown in [33]. Here we assume that the discrete-time model is nonlinear with

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}) + \boldsymbol{w}_k \tag{1}$$

$$\boldsymbol{y}_k = h(\boldsymbol{x}_k, \boldsymbol{\theta}) + \boldsymbol{v}_k \tag{2}$$

where k is the time index, $\boldsymbol{x}_k$ is the state vector, $f(\boldsymbol{x}_{k-1})$ is a nonlinear function, $\boldsymbol{w}_k$ is the process noise, $\boldsymbol{y}_k$ is the measurement, $\boldsymbol{v}_k$ is the measurement noise, $h(\boldsymbol{x}_k, \boldsymbol{\theta})$ is a function of the state vector $\boldsymbol{x}_k$ and the uncertain parameter vector $\boldsymbol{\theta}$. The vector $\boldsymbol{\theta}$ is assumed to be constant throughout the filtering process. The nonlinear functions $f(\boldsymbol{x}_{k-1})$ and $h(\boldsymbol{x}_k, \boldsymbol{\theta})$ are assumed to be continuously differentiable. $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are uncorrelated white noises with zero means and known covariance matrices

$$E\left\{\begin{bmatrix} \boldsymbol{w}_k \\ \boldsymbol{v}_k \end{bmatrix}\begin{bmatrix} \boldsymbol{w}_j^{\mathrm{T}} & \boldsymbol{v}_j^{\mathrm{T}} \end{bmatrix}\right\} = \begin{bmatrix} \boldsymbol{Q}_k \cdot \delta_{kj} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_k \cdot \delta_{kj} \end{bmatrix} \tag{3}$$

where δ_{kj} denotes the Kronecker delta function, which is equal to unity for $k = j$ and zeros elsewhere. $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$ are known symmetric positive definite matrices. This paper focuses on the state estimation for the system with measurement model uncertainty, as uncertain parameters exist in the measurement model of the considered X-ray pulsar-based navigation (XNAV) system. The approach presented here is easy to be extended to estimate the state of the system with dynamic model uncertainty.

For the implementation of the multiple-model (MM) algorithm, a model set should be constructed. For the system shown in (1) and (2), the uncertain parameter vector $\boldsymbol{\theta}$ is approximated by a set of parameter vectors. The parameter set is formed as

$$\boldsymbol{\theta} \in \left\{\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \cdots, \boldsymbol{\theta}^{(M)}\right\} \tag{4}$$

where $\boldsymbol{\theta}^{(\tau)}$ ($\tau = 1, 2, \ldots, M$) is the predetermined element in the parameter set, τ is the model index, M is the total number of the elements. Accordingly, M filters, each depends on a predetermined parameter vector $\boldsymbol{\theta}^{(\tau)}$, run in parallel to estimate the state. To deal with the model nonlinearity, the linearization approach is adopted to implement the parallel filtering. The EKF is derived with the assumption that the linearized system provides close approximation to the true system. The linearization approach is valid for the nonlinear case where the first-order Taylor series approximates the system model effectively. The presented algorithm is expected to be valid as long as the EKF is valid for the problem.

With the initial state estimate $\hat{\boldsymbol{x}}_0^{(\tau)}$, the estimation error covariance $\boldsymbol{P}_0^{(\tau)}$ and the initial weight $\omega_0^{(\tau)}$ for each filter, the RMMAE algorithm updates the estimate $\hat{\boldsymbol{x}}_k^{(\tau)}$, the covariance $\boldsymbol{P}_k^{(\tau)}$ and the weight $\omega_k^{(\tau)}$ recursively. Each cycle of the RMMAE algorithm consists of the following three steps.

Step 1: Parallel filtering

At time k, each filter predicts and updates its state and covariance individually under the assumption that the parameter vector $\boldsymbol{\theta}^{(\tau)}$ matches the true parameter vector $\boldsymbol{\theta}$. For the τ-th filter, the state estimate $\hat{\boldsymbol{x}}_k^{(\tau)}$ and the corresponding error covariance $\boldsymbol{P}_k^{(\tau)}$ are calculated by

$$\hat{\boldsymbol{x}}_{k|k-1}^{(\tau)} = f\left(\hat{\boldsymbol{x}}_{k-1}^{(\tau)}\right) \tag{5}$$

$$\boldsymbol{P}_{k|k-1}^{(\tau)} = \boldsymbol{F}_k\left[\left(\boldsymbol{P}_{k-1}^{(\tau)}\right)^{-1} - \left(\gamma^2 \boldsymbol{P}_{k-1}^{(\tau)}\right)^{-1}\right]^{-1}\boldsymbol{F}_k^{\mathrm{T}} + \boldsymbol{Q}_k \tag{6}$$

$$\hat{\boldsymbol{x}}_k^{(\tau)} = \hat{\boldsymbol{x}}_{k|k-1}^{(\tau)} + \boldsymbol{K}_k^{(\tau)}\left[\boldsymbol{y}_k - h\left(\hat{\boldsymbol{x}}_{k|k-1}^{(\tau)}, \boldsymbol{\theta}^{(\tau)}\right)\right] \tag{7}$$

$$\boldsymbol{K}_k^{(\tau)} = \boldsymbol{P}_{k|k-1}^{(\tau)}\boldsymbol{H}_k^{\mathrm{T}}\left(\theta^{(\tau)}\right)\left[\boldsymbol{H}_k\left(\theta^{(\tau)}\right)\boldsymbol{P}_{k|k-1}^{(\tau)}\boldsymbol{H}_k^{\mathrm{T}}\left(\theta^{(\tau)}\right) + \hat{\boldsymbol{R}}_k\right]^{-1} \tag{8}$$

$$\begin{aligned}\boldsymbol{P}_k^{(\tau)} &= \left[\boldsymbol{I} - \boldsymbol{K}_k^{(\tau)}\boldsymbol{H}_k\left(\theta^{(\tau)}\right)\right]\boldsymbol{P}_{k|k-1}^{(\tau)}\left[\boldsymbol{I} - \boldsymbol{K}_k^{(\tau)}\boldsymbol{H}_k\left(\theta^{(\tau)}\right)\right]^{\mathrm{T}} \\ &\quad + \boldsymbol{K}_k^{(\tau)}\hat{\boldsymbol{R}}_k\boldsymbol{K}_k^{(\tau)\mathrm{T}}\end{aligned} \tag{9}$$

where $\hat{\boldsymbol{x}}_{k|k-1}^{(\tau)}$ is the prediction of the state vector, $\boldsymbol{P}_{k|k-1}^{(\tau)}$ is the corresponding error covariance, $\boldsymbol{K}_k^{(\tau)}$ is the gain matrix, $\boldsymbol{F}_k^{(\tau)} = \frac{\partial f}{\partial \boldsymbol{x}}|_{\boldsymbol{x}=\hat{\boldsymbol{x}}_{k-1}^{(\tau)}}$ and $\boldsymbol{H}_k^{(\tau)} = \frac{\partial h}{\partial \boldsymbol{x}}|_{\boldsymbol{x}=\hat{\boldsymbol{x}}_{k|k-1}^{(\tau)}}$ are the Jacobi matrices. The tuning parameter γ, which has been used for the design of the H_∞ filter in Refs. [6,32,35], gives an upper bound of the energy gain from the uncertainties to the estimation error. If an appropriate parameter γ is found to be such that $\boldsymbol{P}_k^{(\tau)} > 0$, the estimation of an H_∞ filter will remain bounded. In practice, the parameter γ can be set as a large positive constant. $\hat{\boldsymbol{R}}_k \geq \boldsymbol{R}_k$ is a symmetric and positive definite matrix, which is used to take into account the error between the most likely parameter vector in the parameter set and the real parameter vector. Section 4 provides a heuristic method for the design of $\hat{\boldsymbol{R}}_k$.

Step 2: Weight update

The weights of the state estimates obtained from the individual filters are calculated from the residuals $\tilde{\boldsymbol{y}}_k^{(\tau)}$ and the corresponding covariance $\hat{\boldsymbol{\Omega}}_k^{(\tau)}$. The weight for the τ-th filter is calculated as

$$\omega_k^{(\tau)} = \frac{\omega_{k-1}^{(\tau)}\Lambda_k^{(\tau)}}{\sum_{\tau=1}^{M}\omega_{k-1}^{(\tau)}\Lambda_k^{(\tau)}} \tag{10}$$

where

$$\Lambda_k^{(\tau)} = \frac{1}{\sqrt{2\pi\,\hat{\boldsymbol{\Omega}}_k^{(\tau)}}}\exp\left[-\frac{1}{2}\tilde{\boldsymbol{y}}_k^{(\tau)\mathrm{T}}\left(\hat{\boldsymbol{\Omega}}_k^{(\tau)}\right)^{-1}\tilde{\boldsymbol{y}}_k^{(\tau)}\right] \tag{11}$$

$$\tilde{\boldsymbol{y}}_k^{(\tau)} = \boldsymbol{y}_k - h\left(\hat{\boldsymbol{x}}_{k|k-1}^{(\tau)}, \boldsymbol{\theta}^{(\tau)}\right) \tag{12}$$

$$\hat{\boldsymbol{\Omega}}_k^{(\tau)} = \boldsymbol{H}_k\left(\theta^{(\tau)}\right)\boldsymbol{P}_{k|k-1}^{(\tau)}\boldsymbol{H}_k^{\mathrm{T}}\left(\theta^{(\tau)}\right) + \hat{\boldsymbol{R}}_k. \tag{13}$$