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Dynamic stability conditions for a rolling flight vehicle applying continuous actuator

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ABSTRACT

The conditions for dynamic stability of coning motion for a rolling missile are proposed. Considering dynamics of fin actuator, a Proportional–Derivative (PD) controller is applied to the servomechanism employing a permanent magnet DC motor. The necessary and sufficient conditions of dynamic stability are analytically derived. Both steady state and transient responses of actuator are considered in this paper. The differential equation of a whole system for steady state response of actuator leads to a second order differential equation with complex coefficients. A theorem based on Routh–Hurwitz stability criterion is proposed that gives a triple inequality to determine the stability region of PD controller and consequentially, the stability of coning motion. For transient response, the feasibility of a linear matrix inequality should be checked for each PD coefficient. It is shown that considering the transient response of actuator can change the stability region. The controller coefficients that stabilize the coning motion in steady state response may cause instability in transient response of actuator. Finally, validation of this investigation is examined through simulation. Because of continually rolling motion of missile, the cross coupling that is caused by phase lag angle, will occur, so the effect of PD controller coefficients on static phase lag angle is discussed and some technical notes to reduce the cross coupling effect are explained. Furthermore, the influence of roll rate frequency on the region of PD controller coefficients is presented and a safe region for design is recommended.

1. Introduction

There are three maneuver alternatives in aerodynamic control of missiles. Bank-to-turn (BTT), Skid-to-turn (STT) and Rolling airframe (RA). In BTT maneuvering, missiles first roll until the major axis of lifting body is oriented perpendicular to the target line of sight, then maneuvers in pitch [6]. Aircrafts and missiles that require high lifting apply BTT maneuver. A disadvantage of BTT maneuvering is slow maneuvers which reduce the guidance accuracy and increase the miss distance [6]. Most cruciform missiles use STT maneuvering. Applying four canards or tails, a missile without rolling can maneuver in pitch and yaw channels. A benefit of STT maneuvering is fast response to command. But in this maneuver, at least three actuators for pitch, yaw and roll channels are required. Of course in some missiles, roll orientation can be forced to be constant using an open loop control as implemented on the Sidewinder missile, passively with rollerons [1].

Rolling airframe maneuvering is another type of maneuver in which missile provides continuous roll rate. Thus, dynamics of pitch and yaw channels will be replaced by each other and result in lower cost because of the requirement for fewer flight control gyros, accelerometers, and actuators [27]. Lower volume, compensation for dynamic effects of thrust asymmetries and fin misalignments [26], are another advantages of RA maneuvering. Rolling missiles (sometimes it is called spinning missiles) such as Stinger apply RA maneuvering [6].

Coning motion where the nose of the body describes a circle, around the velocity vector [29], is one of the nature characteristics of RA. In some conditions, this coning motion may be unstable. If the coning motion is unstable, the perturbation will become larger gradually and catastrophic flight failure will occur [13]. There are various reasons which lead to an unstable coning motion. Gyroscopic effects [23,17], Magnus effect [28], nonlinear aerodynamic moments [21,19,25], periodic perturbation caused by asymmetries [20], catastrophic yaw caused by roll lock-in resonance [24,15,14] and internal moving component [23,22]. Large amplitude oscillations of cruciform tailed missiles is investigated in [16] in which the cubic aerodynamic model or the linear model can be conser-

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Nomenclature

$C_{l\delta_f}$	roll effectiveness moment coefficient
C_{lp}	roll damping moment coefficient
$C_{m\dot{\alpha}}, C_{mq}$	damping moment derivatives coefficient
$C_{mp\alpha}$	Magnus moment coefficient
$C_{m\delta}$	canard moment coefficient slope
$C_{m\alpha}$	static moment coefficient
$C_{N\alpha}$	normal force coefficient slope
$C_{N\delta}$	canard force coefficient slope
C_x	axis force coefficient
C_h	hinge moment coefficient
F_x, F_y, F_z	axial, side, and normal forces
I_x	axial moment of inertia
I_y	transverse moment of inertia
i	current of DC motor winding
j	$\sqrt{-1}$
J_R, J_L	rotor and load moments of inertia
K_b	back-EMF coefficient of DC motor
K_t	torque constant
K_P, K_D	proportional and derivative coefficients of PD controller
l	reference length
M_x, M_y, M_z	rolling, pitching, and yawing moments
m	mass of missile
N	speed ratio of reducer gear
p, q, r	components of angular velocity in non-rolling frame
P	gyroscopic spin, $I_x p/I_y$
Q	dynamic pressure
$\mathbb{R}$	real numbers set

$\mathcal{R}_1, \mathcal{R}_2$	regions of stability
R, L	resistance and inductance of motor winding
S	reference area
u, v, w	components of velocity in non-rolling frame
V	magnitude of the missile velocity
v_m	input voltage of DC motor (armature voltage)
xyz	body fixed frame Cartesian axes, the x axis along the missile's axis of symmetry
$x\tilde{y}\tilde{z}$	non-rolling Cartesian axes, the x axis along the missile's axis of symmetry
α, β	angles of attack and sideslip in the missile fixed frame
γ_s, γ_d	static and dynamic phase lag angle
δ	complex angle of actuator deflection
δ_y, δ_z	actuator deflection angles
δ_r, δ_c	reference and canards deflection angles
δ_f	fin cant angle
ξ	complex angle of attack in the non-rolling system, $\beta + j\alpha$
$\tilde{\xi}$	complex angle of attack in the missile-fixed system, $\tilde{\beta} + j\tilde{\alpha}$
τ	output moment of motor
τ_L	load moment
ϕ	roll angle of missile
φ	output roll angle of DC motor

Superscripts

$\dot{()}$	time derivative
$\tilde{()}$	non-rolling airframe

vatively used to obtain the expression of the minimum fin cant and steady-state spin that assure the catastrophic yaw avoidance.

Ref. [32] has introduced another source of unstable coning motion for rolling missiles. Taking into account the dynamics of the fin actuators, it is shown that the hinge moment may lead to the instability of rolling missiles under specific conditions.

The actuators of rolling missiles, from a performance point of view, are classified into On-Off or two positions types and continuous types. The servomechanism of a rolling missile with continuous actuator is consisted of a permanent magnet DC motor, a PD controller, a speed reducer gear, canards and a potentiometer to measure the canard deflection. Analysis and design of a continuous actuator which leads to stable coning motion due to hinge moment are investigated in this paper.

The analysis and design of an autopilot for a rolling missile applying continuous actuator were conducted by Lestage [9] and Malmgren [12]. Using describing function method, the mechanism of the limit circular motion induced by the backlash of two position actuators which is generated from the gear drive and its nonlinearity effects are studied in [33]. A comparison between the performance of two position and continuous actuators and choice of aerodynamic force function according to roll angle of vehicle is presented in [8]. Nobahari and Mohammadkarimi [26] developed an autopilot for a rolling missile implementing on-off actuator based on linear control theory and multiple describing function technique. Attitude control for rolling missiles with separated pitch and yaw control were implemented by Creagh and Mee [4]. However these efforts extend the separated channel design method for non-rolling missiles to rolling missiles, while cross coupling effects would not considered. Yan et al. have studied the stability of coning motion for rolling missiles with a rate loop [30], an attitude autopilot [31] and acceleration autopilot [10] and presented the stability criterion analytically. Hinge moment effect to instability of coning motion was investigated by Zhou et al. [32]. The control parameters that are presented to analysis of the stability condition of coning motion in these references, are one dimensional. It means, they employed a proportional controller (gain) to stabilize the coning motion. Although Zhou et al. [32], have exploited a PD controller (two dimensional parameters), a parameter so-called "stiffness coefficient" has analyzed, however the effect of derivative term is not shown.

In this paper, it is shown that a proportional term lonely cannot decrease the phase lag angle as minimum as possible and a derivative term is required. Therefore, it can be implemented in servo dynamics.

The stability criteria for coning motion of rolling missiles employing classical three-loop autopilot, and apply proportional controller are derived by Li et al. [11]. However, no comment is proposed for which controller's gain may compensate actuator or measurement time delay or consequently pitch–yaw channel cross coupling.

The objective of this paper is to establish the design and implementation conditions for a rolling missile and to stabilize its coning motions applying continuous actuator. The continuous actuator is composed of a permanent DC motor that can be controlled with armature voltage. After modeling the dynamics of missile and actuator in Section 2, Section 3 determines some stability region analytically, for PD controller coefficients both for steady state and transient responses of actuator. In the simulation results in Section 4, there are examples of controller parameters that yield stable, critical and unstable coning motions. Moreover, for a constant stiffness coefficient, it is shown that, as the derivative coefficient increases, spectrum density of canards deflection approaches to high frequencies. The time delay as phase lag angle which has an important role on cross coupling is calculated in stability region of servo parameters. In addition, the effect of roll rate frequency on