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Isentropic wave propagation in a viscous fluid moving along a lined duct with shear flow and its implication for ultrasonic flow measurement

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ABSTRACT

Present paper investigates the influences of the fluid viscosity and acoustic impedance of the wall on the isentropic acoustic wave propagating along a shear flow contained in a circular pipeline. On the assumption of an axisymmetric acoustic wave, mathematical formulation with respect to the acoustic velocity is deduced from the conservation of mass and momentum. Numerical calculation is processed through an iterative procedure based on the Fourier–Bessel theory. Comparisons of the wave attenuation and measurement performance of an ultrasonic flow meter are addressed among the rigid, steel and aluminum walls. Meanwhile, the differences between the laminar and turbulent flow are provided. In conclusion, parametric analyses of the influence of the acoustic impedance on the wave attenuation and ultrasonic measurement performance are given.

1. Introduction

Propellant gauging requirements for the management of fluid in the aerospace applications and the resupply of a wide variety of fluids to the on-orbit platforms have stimulated intensive research on the novel propellant gauging methods [25,5]. The ultrasonic flow meter can provide non-invasive no-moving-parts construction, offer high potentials for measuring rapidly varying flow velocities and impose no impedance on the flow [19]. Recently, Matthijsen and Put [20] developed an ultrasonic flow meter for the space application under ESA development program.

Based on the inviscid fluid assumption, Lechner [18] investigated the effect of the shear mean flow on the performance of the ultrasonic flow meter, Willatzen [29] made constructive comments on the work of Lechner. Furthermore, comparisons of different mathematical models of wave propagation in the inviscid fluid were carried out by Willatzen [31]. The effects of the temperature [30,6] and the acoustic impedance of the wall [32] on the measurement performance of the ultrasonic flow meter were analyzed thereafter.

In the above-mentioned contributions, the ignorance of the fluid viscosity [10] and acoustic absorption of the wall [3,12,26] leads to the failure of the description of the wave attenuation. In the rigid-walled pipeline, Peat [23] was the first to analyze the fundamental acoustic mode in the perfect gas considering the effects of the viscous dissipation and thermal conduction in the presence of a parabolic mean flow. In the case of a stationary liquid, Elvira-Segura [11] adopted the isentropic acoustic assumption neglecting the effect of the thermal conduction. Recently, the authors [7] expanded the research of the isentropic acoustic wave to the condition that a uniform mean flow is present.

If the wall is not rigid, the corresponding acoustic impedance may alter the features of the acoustic wave. In the framework of an inviscid fluid, the Ingard–Myers boundary condition [22] was widely used to handle the influence of the acoustic impedance on wave propagation. However, such method neglected the viscous dissipation in the fluid. By analyzing the acoustic wave propagating in a viscous boundary layer, different improved models of the boundary condition were proposed by Brambley et al. [4], Rienstra and Darau [27], and Auregan et al. [2,24], to mention a few. Although the viscous dissipation was taken into consideration at the viscous boundary layer, the governing equation of wave propagation was established in the framework of the inviscid fluid.

Present paper tends to simultaneously analyze the effects of the acoustic impedance and fluid viscosity on wave propagation

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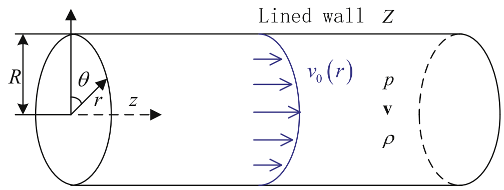

Fig. 1. (Color online.) Configuration of the problem in the circular cylindrical pipeline.

in the presence of a shear mean flow (laminar and turbulent flow). Theoretical analysis is based on the Fourier–Bessel theory [28,17] which was used in the previous study [6–8]. As particular attention is imposed on the acoustic features in the liquid flow, the isentropic acoustic assumption, on the neglect of the thermal conduction adopted from Elvira-Segura [11] and Chen et al. [7], is presumed. Furthermore, the numerical study concentrates on the wave attenuation and measurement performance of the ultrasonic flow measurement.

2. Mathematical description of problem

In this section, the convected wave propagation through a shear mean flow is investigated in the circular cylindrical coordinate system as shown in Fig. 1. r, θ, and z are the radial, circumferential, and axial directions respectively. R is the pipeline radius and $v_0(r)$ is the shear flow profile. Z represents the acoustic impedance of the wall.

In what follows, an acoustic wave is assumed to propagate in a steady viscous fluid with a shear mean flow. The disturbances to the fluid pressure (p), flow velocity ($\mathbf{v}$) and density (ρ) are supposed to be small enough to satisfy the linear approximations. With no effect of a swirl flow, the present study concentrates on the characteristics of the axisymmetric acoustic wave under the influences of the fluid viscosity and acoustic impedance of the wall. On the neglect of the thermal conduction, the acoustic wave is simplified to be isentropic.

2.1. Governing equation

In the case of an isentropic acoustic wave, mathematical formulations start from the conservation of mass and momentum while the energy conservation is omitted

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0,$$
$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla p + \eta\nabla^2\mathbf{v} + \left(\zeta + \frac{\eta}{3}\right)\nabla(\nabla\cdot\mathbf{v}), \tag{1}$$

where η and ζ are the coefficients of the shear and bulk viscosity [11]. As in the literature, the disturbances to the fluid viscosities due to wave propagation are not considered [11,7]. Thus, the physical variables (density, velocity and pressure) are divided into the time-averaged mean quantities (ρ_0, $\mathbf{v}_0$ and p_0) and the first-order acoustic quantities (ρ', $\mathbf{v}'$ and p'). On the assumption of the uniform steady density ($\rho_0 = \text{constant}$) and shear mean flow ($\mathbf{v}_0 = [0, 0, v_0(r)]$ in the cylindrical coordinate as shown in Fig. 1), one obtains

$$\nabla\cdot\mathbf{v}_0 = 0,$$
$$\rho_0(\mathbf{v}_0\cdot\nabla)\mathbf{v}_0 = -\nabla p_0 + \eta\nabla^2\mathbf{v}_0 + \left(\zeta + \frac{\eta}{3}\right)\nabla(\nabla\cdot\mathbf{v}_0) = 0. \tag{2}$$

Due to the linear acoustic approximations, the Taylor's first-order expansions of Eq. (1) can be written by

$$\frac{\partial \rho'}{\partial t} + (\mathbf{v}_0\cdot\nabla)\rho' + \rho_0\nabla\cdot\mathbf{v}' = 0, \tag{3}$$
$$\rho_0\left[\frac{\partial \mathbf{v}'}{\partial t} + (\mathbf{v}_0\cdot\nabla)\mathbf{v}' + (\mathbf{v}'\cdot\nabla)\mathbf{v}_0\right] = -\nabla p' + \eta\nabla^2\mathbf{v}' + \left(\zeta + \frac{\eta}{3}\right)\nabla(\nabla\cdot\mathbf{v}'). \tag{4}$$

The assumption of an isentropic acoustic wave leads to the constraint of $p' = c_0^2\rho'$ where c_0 is the adiabatic sound speed which is assumed to be constant. Eq. (3) then may be simplified to

$$\frac{\partial p'}{\partial t} + (\mathbf{v}_0\cdot\nabla)p' + \rho_0 c_0^2\nabla\cdot\mathbf{v}' = 0. \tag{5}$$

As the acoustic perturbations are considered to be monochromatic and axisymmetric, the fluctuating quantities can be partly expressed as $\exp[\mathrm{i}(\omega t - k_0 K z)]$, where ω, K and $k_0 = \omega/c_0$ are the angular frequency, the dimensionless axial wavenumber and the inviscid total wavenumber respectively. To give a dimensionless expression, the local Mach number ($M(r) = v_0(r)/c_0$) is introduced to describe the shear flow profile, then Eqs. (5) and (4) can be deduced to

$$\mathrm{i}\omega(1-KM)p' + \rho_0 c_0^2\nabla\cdot\mathbf{v}' = 0 \quad\Rightarrow\quad p' = -\frac{\rho_0 c_0^2}{\mathrm{i}\omega(1-KM)}\nabla\cdot\mathbf{v}', \tag{6}$$
$$\mathrm{i}\omega(1-KM)\mathbf{v}' + (\mathbf{v}'\cdot\nabla)\mathbf{v}_0 = -\frac{\nabla p'}{\rho_0} + \frac{\eta}{\rho_0}\nabla^2\mathbf{v}' + \frac{1}{\rho_0}\left(\zeta + \frac{\eta}{3}\right)\nabla(\nabla\cdot\mathbf{v}'). \tag{7}$$

Substituting Eq. (6) into Eq. (7) yields

$$\mathrm{i}\omega(1-KM)\mathbf{v}' + (\mathbf{v}'\cdot\nabla)\mathbf{v}_0 = \nabla\left[\frac{c_0^2}{\mathrm{i}\omega(1-KM)}\right]\nabla\cdot\mathbf{v}' + \frac{\eta}{\rho_0}\nabla^2\mathbf{v}' + \left[\frac{c_0^2}{\mathrm{i}\omega(1-KM)} + \frac{1}{\rho_0}\left(\zeta + \frac{\eta}{3}\right)\right]\nabla(\nabla\cdot\mathbf{v}'). \tag{8}$$

In the case of an axisymmetric acoustic wave, the circumferential component of the acoustic velocity can be neglected, then the acoustic velocity can be written as $\mathbf{v}' = [v_r', v_z']$. Expanding Eq. (8) in the cylindrical coordinate system [1] yields

$$\begin{aligned}\mathrm{i}\omega R^2(1-KM)v_r' &= \frac{c_0^2 K}{\mathrm{i}\omega(1-KM)^2}\frac{\mathrm{d}M}{\mathrm{d}x}\left[\frac{1}{x}\frac{\partial}{\partial x}(x v_r') - \mathrm{i}k_0 R K v_z'\right] + \frac{\eta}{\rho_0}\left[\frac{\partial}{x\partial x}\left(x\frac{\partial v_r'}{\partial x}\right) - \frac{1}{x^2}v_r' - k_0^2R^2K^2 v_r'\right] \\ &\quad + \left[\frac{c_0^2}{\mathrm{i}\omega(1-KM)} + \frac{1}{\rho_0}\left(\zeta + \frac{\eta}{3}\right)\right]\frac{\partial}{\partial x} \times \left[\frac{1}{x}\frac{\partial}{\partial x}(x v_r') - \mathrm{i}k_0 R K v_z'\right],\end{aligned} \tag{9}$$

$$\begin{aligned}\mathrm{i}\omega R^2(1-KM)v_z' + c_0 R\frac{\mathrm{d}M}{\mathrm{d}x}v_r' &= \frac{\eta}{\rho_0}\left[\frac{\partial}{x\partial x}\left(x\frac{\partial v_z'}{\partial x}\right) - k_0^2R^2K^2 v_z'\right] \\ &\quad - \mathrm{i}k_0 R K\left[\frac{c_0^2}{\mathrm{i}\omega(1-KM)} + \frac{1}{\rho_0}\left(\zeta + \frac{\eta}{3}\right)\right] \times \left[\frac{1}{x}\frac{\partial}{\partial x}(x v_r') - \mathrm{i}k_0 R K v_z'\right],\end{aligned} \tag{10}$$