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Optimal uses of reaction wheels in the pyramid configuration using a new minimum infinity-norm solution

Hyungjoo Yoon*, Hyun Ho Seo, Hong-Taek Choi

Satellite Control System Department, Korea Aerospace Research Institute, Daejeon 305-806, Republic of Korea

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ABSTRACT

In this study, simple methods are presented to improve the agility performance of a spacecraft with four reaction wheels in the pyramid configuration. A new and simple method is proposed to determine the momentum and the torque envelopes, which are defined as the maximum momentum and torque capacities of the wheel array, respectively. Then, based on the geometry of the envelopes, the best shape of the pyramid configuration needed to deliver optimal agility performance is discussed. In this paper, new methods are also proposed to optimally distribute three-dimensional torque and momentum commands, to the individual reaction wheels. The developed methods are based on the use of novel algorithms to solve minimum infinity-norm, or L_∞-norm, problems. These algorithms can easily be implemented with minimal modification of conventional ones, but yield considerable improvement of agility performance in numerical examples.

1. Introduction

Agility performance has become one of the key factors in developing/operating modern satellite systems, especially for Earth-imaging satellites, because it determines the number of available imaging targets within the duration of a given pass. Agility performance can be improved in various ways, but it is mainly determined by the maximum torque and momentum capacities of the actuators. A modern satellite is generally equipped with an array of at least three, possibly more, reaction wheels for redundancy. Hence, the total combined capacity of the array, which is referred to as an 'envelope', should be considered. This envelope is determined not only by the capacity of individual wheels, but also by the configuration of the wheel array. In this paper, we discuss how to determine the wheel configuration needed for optimal agility performance.

Another closely related problem that should be considered, is the efficient distribution of three-dimensional torque and momentum commands to individual reaction wheels. Because there are generally more than three reaction wheels, it becomes necessary to solve an under-determined linear-equations system, which in general has an infinite number of solutions.

In fact, this problem can be considered a special case of the control allocation problem. There have been a lot of investigations of control allocation problems for aircraft (see Ref. [7] and the references therein) and a few on spacecraft attitude control (e.g., see Ref. [3]). However, the focus of these studies was not the best use of wheel array capacity to achieve optimal agility, which is the main topic of this paper.

Conventionally, the minimum L_2-norm solution, which minimizes the square sum of the individual torque/momentum, is used because it minimizes the total power/energy of the wheel array. However, as will be shown later, this method does not fully utilize the envelopes of the wheel array. On the other hand, the minimum L_∞-norm (or 'infinity-norm') solution, which minimizes the maximum absolute value of the individual torque/momentum, may be a better choice for higher agility performance. While the minimum L_2-norm solution can easily be obtained using a pseudo-inverse matrix, it is well known that the minimum L_∞-norm solution cannot be expressed in a simple closed form, and thus needs more sophisticated algorithms to 'search' for it.

Cadzow proposed just such an algorithm [1,2]. His algorithm is efficient and is applicable to under-determined problems in any number of dimensions, but it is subject to the Haar condition [5]. Moreover, the algorithm does not solve the problem itself but, in fact, solves its 'dual optimization problem'. For this reason, it does not help much to understand the nature of the problem. Gravagne and Walker [4] proposed an algorithm also based on the dual problem, and showed how the solution could be applied to multi-link robot controls. Markley et al. [8] first related the minimum

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* Corresponding author. Tel.: +82 10 3324 3660.
E-mail addresses: drake.yoon@gmail.com (H. Yoon), seo2h@kari.re.kr (H.H. Seo), hongtaek@kari.re.kr (H.-T. Choi).

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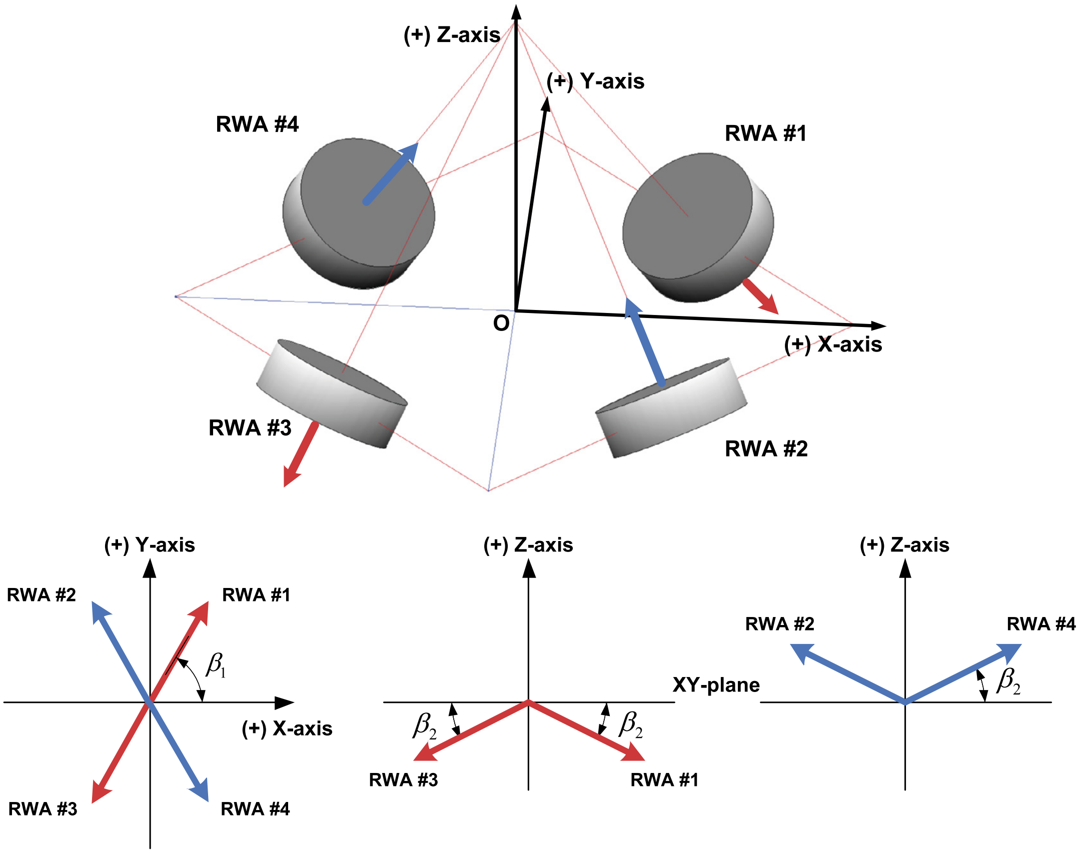

Fig. 1. Reaction wheel array in the pyramid configuration.

infinity-norm problem to spacecraft attitude control with reaction wheels, and provided the motivation for the present paper. They presented the results from a study of the nature of the torque and momentum envelopes, and provided a scheme to define them. They also proposed an attitude control loop based on their minimum infinity-norm solution algorithm. Their paper clearly revealed the geometric aspects of the envelopes, and their results are widely applicable to general configuration (e.g., in terms of number of wheels, sizes, and axis directions) of a reaction wheel array. Their method, however, is based upon a purely geometric approach without considering coding efficiency, meaning that there is room for improvement in terms of coding and computational efficiency. Verbin and Ben-Asher [9] also proposed an algorithm for a case with four reaction wheels.

In this paper, simpler and computationally more efficient methods are proposed than those presented in the earlier works [8, 9]. Our works is focused on the pyramid configuration with four identical reaction wheels, which is in fact the industry standard owing to its minimum number of redundancies and its symmetric capacities. We propose a simple new method which defines the momentum/torque envelopes. Then we provide a scheme to optimize the pyramid configuration for the inertia properties of a given spacecraft, using the relationship between its moment of inertia, and the envelope under consideration. For the distribution of the torque/momentum, we herein propose a new algorithm to obtain the minimum infinity-norm solution, that also runs much faster than that of Ref. [8].

Another distinct feature of the present work is that it provides another new algorithm which calculates an optimal momentum distribution with the wheel speeds minimally diverging from a nominal set value, in the sense of the L_∞-norm distance. This algorithm can be used to make the wheel speeds stay as close to the (non-zero) nominal speeds as possible, even when the total angular momentum of the array is zero. This feature is useful in practice to keep the wheels from operating at, or crossing zero rpm (at which wheel characteristics become nonlinear due to static and Coulomb friction). This nonlinear behavior near zero rpm may instantaneously increase the attitude control error and degrade the mechanical and electrical reliability of the wheel. So, in some space programs, it is preferable to avoid the zero rpm operation completely, if possible.

Finally, comparative numerical simulations are provided to show the effectiveness of the proposed methods. It will be shown that a control loop using the proposed methods, yields superior agility performance over the conventional L_2-norm method, and also successfully leads the wheels to a given non-zero nominal speed after completing the maneuver.

2. Momentum and torque envelope

2.1. The pyramid configuration

In this section, we propose a new means of composing the momentum and the torque envelopes of a reaction wheel array. This paper mainly deals with a four-wheel array in a pyramid configuration of the type shown in Fig. 1, which is the most common in practice. (The direction of each spin axis can be flipped, but it is defined intentionally, as shown in Fig. 1, to make the null space vector according to Eq. (4). The reason will be explained in Section 4.3.)

Let us denote the wheel spin direction vectors by $W = [\hat{w}_1, \cdots, \hat{w}_4]_{3\times4}$; the total angular momentum $\mathbf{H}_\mathrm{t} \in R^3$ and the angular momenta vector of the array $\mathbf{H}_\mathrm{w} = [H_1, \cdots, H_4]^T \in R^4$ are then related according to