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Robust recursive filtering for uncertain systems with finite-step correlated noises, stochastic nonlinearities and autocorrelated missing measurements

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ABSTRACT

In this paper, the robust recursive filtering problem is studied for a class of uncertain systems with finite-step correlated noises, stochastic nonlinearities and autocorrelated missing measurements. The correlated noises and stochastic nonlinearities are simultaneously considered, where process noises and measurement noises are arbitrary finite-step autocorrelated and cross-correlated. The missing measurements appear in a random way which is governed by missing rates obeying a certain probability distribution. The autocorrelation of missing rates, for the first time, is introduced to reflect the interaction of network bandwidth at adjacent sampling times. The aim of the addressed filtering problem is to design an unbiased robust recursive filter such that, for the uncertain systems, the filtering error is minimized at each sampling time. It is shown that the filter gain is obtained by solving a recursive matrix equation. A numerical simulation example is presented to illustrate the effectiveness of the proposed algorithm.

1. Introduction

In the past few decades, Kalman filtering algorithm has attracted much attention for its simple structure and good performance [1], and it has been widely used in the field of engineering such as communication, navigation and target tracking [4,12,21]. However, when system model contains parameter uncertainty, the filtering performance deteriorates inevitably. In order to deal with this problem, much work has been done to design robust filter against the adverse influence of parameter uncertainty [2,3,5,6,14,25,26,33]. The results for systems with random parameter uncertainty are reported in [3,14,25]. In [2], the robust Kalman filtering problem is investigated for uncertain state delay systems with random observation delays. Furthermore, H_∞ filtering algorithms are proposed in [5,6,26] for discrete uncertain systems with bounded perturbation, and the H_∞ filter is designed in [33] for continuous-time linear systems with parameter uncertainty that belongs to a given convex bounded polyhedral domain.

The correlated noises are encountered in systems such as the radar tracking system where the process noises and measurement noises are cross-correlated [24]. The systems with correlated noises have received constant attention and a series of results have been published [22,23,30]. As a typical case, the one-step correlation is assumed to reflect the interaction of noises at different sampling times, which causes the correlation between system state and noises [7,10,16]. Ref. [8] proposes robust filter for uncertain dynamical system with finite-step correlated process noises. The optimal robust non-fragile Kalman-type recursive filtering is proposed in [9] for systems with finite-step autocorrelated measurement noises.

The nonlinear phenomena are common in practice. If the system involves serious nonlinearities, it is difficult to design a filter with good performance only considering the linear model. Therefore, it is necessary to draw the nonlinear analysis into the filter design [5,11,20,26,31]. In recent years, the stochastic nonlinearities, containing white noises in the nonlinear functions, have attracted many research interests. For example, the robust H_∞ finite-horizon filtering problem is solved in [6] for uncertain nonlinear stochastic systems; Ref. [29] is concerned with the filtering problem for stochastic nonlinear time-delay systems; Ref. [32] proposes state estimator satisfying the criteria to suppress stochastic nonlinearities; the extended Kalman filtering problem is investigated in [15] for stochastic nonlinear systems. However, up to now, the state estimation for systems with stochastic nonlinearities as well as the arbitrary finite-step autocorrelated and cross-correlated noises has not received adequate research due probably to its complex derivation.

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In sensor network systems, the missing measurements frequently appear and generally become one of the major constraints for the system performance [18,27]. When communication is not established perfectly due to limited bandwidth of network, the missing measurements appear with a random missing rate for each measurement channel. The missing rates in [2,3,5,14] are regarded as Bernoulli random variables that measurements are completely received or missed. The filtering problem for complex network systems with Bernoulli missing rates and time delays is studied in [34]. With further extension, in [6,16,29], the missing rate for each sensor is governed by an individual random variable obeying a certain probability distribution that the Bernoulli distribution is a special case. On the other hand, the measurement missing rates can be autocorrelated at consecutive sampling times, since network congestion appears at one sampling time and the bandwidth of network need several subsequent sampling times to recover. Unfortunately, this phenomenon has not received adequate attention in the existing literature, not to mention the situation when parameter uncertainty, finite-step correlated noises and stochastic nonlinearities simultaneously appear in the system.

Based on the above discussion, in this paper, we aim to investigate a class of uncertain systems with finite-step autocorrelated and cross-correlated noises, stochastic nonlinearities and autocorrelated missing measurements. The expression of system uncertainties covers both dependent and independent parameter perturbations. The stochastic nonlinearities are defined by mean properties and represent several kinds of well-studied nonlinear functions. The process noises and measurement noises are arbitrary finite-step autocorrelated and cross-correlated, where the universal assumptions are capable of covering any finite-step cases of noise correlation. The missing measurements are determined by missing rates obeying a certain probability distribution, and the additional autocorrelation of missing rates represents the interaction of fluctuated network bandwidth at adjacent intervals. The main contributions of this paper are emphasized as follows: 1) the assumptions of arbitrary finite-step autocorrelation and cross-correlation are universal; 2) the autocorrelation of missing rates, for the first time, is introduced to describe the interaction of network bandwidth; 3) the stochastic nonlinearities are included in system model, thereby better reflecting the reality; 4) the proposed robust recursive filtering algorithm is unbiased and suitable for online applications.

The remainder of this paper is organized as follows. In Section 2, the filtering problem is formulated for a class of uncertain systems with finite-step correlated noises, stochastic nonlinearities and autocorrelated missing measurements. The main results of this paper are showed in Section 3. In Section 4, a numerical simulation example is presented to illustrate the effectiveness of the novel algorithm. This paper is concluded in Section 5.

Notations. The notations in this paper are standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ matrices, respectively. $E\{x\}$ is the expectation of random variable x and $Cov[\alpha,\beta]$ denotes the covariance of random vectors α and β. P^T and $P^\dagger$ are the transpose and Moore–Penrose pseudo inverse of matrix P. $P>0$ means P is real, symmetric and positive definite. $tr(\cdot)$ stands for the trace of a matrix and $diag(a_1,\dots,a_r)$ denotes a diagonal matrix whose diagonal entries are $a_1,\dots,a_r$. I and 0 represent the identity matrix and the zero matrix with appropriate dimensions, respectively. If the dimensions of matrices are not explicitly stated, it is assumed to be compatible for algebraic operations.

2. Problem formulations

In this paper, we consider the following class of uncertain systems with finite-step correlated noises, stochastic nonlinearities and autocorrelated missing measurements:

$$x_{k+1} = A_k x_k + f(x_k,\varepsilon_k) + B_k\omega_k \tag{1}$$

$$y_k = \Xi_k C_k x_k + D_k v_k \tag{2}$$

where

$$\begin{cases} A_k = \bar{A}_k + \sum_{i=1}^{m} A_{i,k}\zeta_{i,k} \\ B_k = \bar{B}_k + \sum_{i=1}^{m} B_{i,k}\zeta_{i,k} \\ C_k = \bar{C}_k + \sum_{i=1}^{m} C_{i,k}\zeta_{i,k} \\ D_k = \bar{D}_k + \sum_{i=1}^{m} D_{i,k}\zeta_{i,k} \end{cases} \tag{3}$$

$x_k\in\mathbb{R}^n$ is the state to be estimated, $y_k\in\mathbb{R}^q$ is the measurement output, $\omega_k\in\mathbb{R}^p$ is the process noise and $v_k\in\mathbb{R}^r$ is the measurement noise. $\bar{A}_k\in\mathbb{R}^{n\times n}$, $\bar{B}_k\in\mathbb{R}^{n\times p}$, $\bar{C}_k\in\mathbb{R}^{q\times n}$, and $\bar{D}_k\in\mathbb{R}^{q\times r}$ are known deterministic matrices, ε_k is a zero-mean white noise uncorrelated with other noises. Random sequences $\zeta_{i,k}$ $(i=1,2,\dots,m)$ are introduced for describing the parameter uncertainty, and they are zero-mean with $E\{\zeta_{i,k}\zeta_{j,l}\}=\delta_{i,j}\delta_{k,l}$, where δ is the Dirac Delta function. $A_{i,k}\in\mathbb{R}^{n\times n}$, $B_{i,k}\in\mathbb{R}^{n\times p}$, $C_{i,k}\in\mathbb{R}^{q\times n}$, and $D_{i,k}\in\mathbb{R}^{q\times r}$ are known and signify the directions of parameter perturbations.

The initial state x_0, process noise ω_k, and measurement noise v_k have following statistical properties:

$$E\{x_0\}=\bar{x}_0,\qquad E\left\{(x_0-\bar{x}_0)(x_0-\bar{x}_0)^T\right\}=P_0 \tag{4}$$

$$E\{\omega_k\}=0,\qquad E\{v_k\}=0 \tag{5}$$

$$E\left\{\omega_k\omega_l^T\right\} = Q_k\delta_{k,l} + \sum_{t=1}^{g_k} Q_{k,l}(\delta_{k,l+t}+\delta_{k,l-t}) \tag{6}$$

$$E\left\{v_k v_l^T\right\} = R_k\delta_{k,l} + \sum_{t=1}^{g_k} R_{k,l}(\delta_{k,l+t}+\delta_{k,l-t}) \tag{7}$$

$$E\left\{\omega_k v_l^T\right\} = S_k\delta_{k,l} + \sum_{t=1}^{g_k} S_{k,l}(\delta_{k,l+t}+\delta_{k,l-t}) \tag{8}$$

where $P_0>0$, $Q_k>0$ and $R_k>0$, g_k represents the step number of correlation, $Q_{k,l}$, $R_{k,l}$ and $S_{k,l}$ are known matrices with appropriate dimensions.

Remark 1. The process noise ω_k and measurement noise v_k are g_k-step autocorrelated and cross-correlated. In practice, the process noise ω_k and the measurement noise v_k maybe have respective correlated step numbers. For example, ω_k and v_k are f_k^{ω}-step forward, b_k^{ω}-step backward and f_k^{v}-step forward, b_k^{v}-step backward autocorrelated respectively, and they are $f_k^{\omega,v}$-step forward, $b_k^{\omega,v}$-step backward cross-correlated. Then, the step number g_k can be determined by $\max\{f_k^{\omega},b_k^{\omega},f_k^{v},b_k^{v},f_k^{\omega,v},b_k^{\omega,v}\}$. Therefore, the more general assumptions (6)–(8) are applicable to the noises with arbitrary finite-step autocorrelation and cross-correlation, and can cover the various attributes of correlations concerned in the majority of existing literature.