Aerospace Science and Technology 39 (2014) 481–490

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

A discrete adjoint harmonic balance method for turbomachinery shape optimization

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ARTICLE INFO

Article history:
Received 15 October 2013
Received in revised form 5 May 2014
Accepted 12 May 2014
Available online 1 July 2014

Keywords:
Harmonic balance
Discrete adjoint method
Aerodynamic shape optimization
Turbomachinery
Aeroelasticity
Limit-cycle oscillations

ABSTRACT

The harmonic balance method has seen an increasing popularity in the solution of time-periodic problems because of its computational efficiency and its ability to model dynamically nonlinear fluid phenomena. In addition, the mathematically steady nature of this technique makes it ideal for adjoint sensitivity analysis of unsteady problems. In this work, a novel optimization framework consisting of three components: a harmonic balance based unsteady cascade flow solver, an accompanying adjoint solver and a quasi-Newton optimization solver; have been developed. The discrete adjoint solver is obtained with the aid of an automatic differentiation tool, TAPENADE. To demonstrate the efficiency and accuracy of the method, we present shape optimization and adjoint sensitivity computations for a two-dimensional compressor cascade. Steady inverse design of this cascade is performed to investigate the effects of two shape parameterization methods, namely Hicks–Henne bump functions and mesh points. Shape optimization is performed to improve the aerodynamic damping characteristics of a vibrating cascade row. In addition, the unsteady adjoint technique is used to determine the frequency of vibration that would drive the system to limit-cycle, which defines the stability limit of the cascade.

1. Introduction

With the advancement of computer technology and computational fluid dynamic (CFD) algorithms in the past couple of decades, computational design optimization has become a prominent research emphasis. Traditionally, investigators have used two families of optimization techniques, namely stochastic and deterministic methods in aerodynamic design optimization. Examples of stochastic methods include genetic algorithms (GAs) and simulated annealing (SA) algorithms, which are capable of determining global minimum points and are easy to implement in well-established CFD codes [50]. However, these approaches tend to be computationally expensive as they generally require a very large number of CFD computations. Gradient-based deterministic optimization techniques, on the other hand, can be more difficult to develop but are generally more efficient. In the gradient-based approach, which includes techniques like tangent-linear and adjoint, it is critical to evaluate the sensitivity (of an objective function to design variables) in an accurate and efficient manner. When the number of design variables is large, the adjoint (reverse) method is preferred due to its efficiency compared to the tangent-linear (forward) method.

The adjoint method was first used in fluid mechanics by Pironneau [37]. Later on, Jameson [22] extended the adjoint method to Euler-based airfoil and wing shape optimization. Following Jameson's pivotal work, many investigators have adopted this technique in the field of aerodynamic shape optimization [1,22,25,27,31,35,42]. In his original work, Jameson derived the adjoint equations analytically and then discretized them to compute the sensitivities. That approach is referred to as the continuous adjoint method. In the other approach, which is called the discrete adjoint method, the governing equations (Navier–Stokes or Euler) are discretized first and then adjoint solver is derived from the CFD solver. Anderson and Bonhaus [1] and Nielsen and Anderson [35] hand-coded a discrete viscous adjoint solver for sensitivity analysis. In general, linearization of the turbulence models by hand is very difficult and time-consuming and thus the majority of the early adjoint researches, either continuous or discrete, have assumed the turbulent viscosity to be "frozen" [1,25,35]. Recently, Lyu et al. [29] employed an automatic differentiation tool, TAPENADE [17], to develop a discrete adjoint solver for the RANS equations in which the Spalart–Allmaras turbulence model [40] was used. In that work, they compared the accuracy of sensitivities obtained by frozen turbulence and "full" turbulence and concluded

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http://dx.doi.org/10.1016/j.ast.2014.05.015
1270-9638/

that full turbulence adjoint computations are needed for more accurate results.

Generally, the adjoint methods propagate the sensitivities of the objective function to design variables by a series of calculations, which are performed in the reverse order of the nominal CFD computations. During these calculations, the gradients at that particular step are propagated backward, and the final gradient information will be a combination of these propagated partial derivatives. The reverse adjoint calculation normally requires the entire time history of the flow computation to be stored, which may be extremely costly for a time-accurate computation. Because of this, the majority of the research on the application of the adjoint sensitivities in aerodynamic design optimization has focused on steady flows. Recently, there have been attempts to apply the method to unsteady time-domain [33,47]. In order to minimize the storage requirements of the time-accurate adjoint computations, a variety of modifications were developed. For example, Wang et al. [48] developed a checkpointing algorithm, which significantly reduces the data storage at the expense of increased computational cost. Nadarajah and Jameson [33] proposed to store only one period of the solution in their adjoint computation. Beran et al. [3] developed a technique in which they used proper orthogonal decomposition (POD) to compress the history of time-accurate computations greatly decreasing the storage requirements.

In the past decade, the mixed-time/frequency domain harmonic balance method of Hall et al. [16] has been used for the solution of various time-periodic problems [7–9,14,15,20,41]. In this approach, the time-periodic flow field is modeled as a number of sub-time level solutions that span the entire temporal period of excitation. These sub-time level solutions, which are solved simultaneously, are coupled through a pseudo-spectral approximation of the time-derivative term in the Euler (or the Navier–Stokes) equations. With the use of the pseudo-spectral operator, the explicit time dependence in the governing equations vanishes and the problem becomes mathematically steady. Therefore, compared to a time-accurate adjoint approach, the harmonic balance method has the added advantage of not having to store any time history of the solution. In addition, the adjoint harmonic code can use convergence acceleration techniques (multigrid, etc.) to speed-up convergence. Typically, the convergence rates for the nominal and the discrete adjoint codes should be identical. Because of these added advantages, investigators have recently developed adjoint solvers based on nonlinear harmonic balance codes [30,34,42]. For example, Thomas et al. [42] developed a discrete adjoint harmonic balance solver with the aid of a commercial automatic differentiation (AD) tool, TAF [12,13] and performed a sensitivity analysis for a pitching NLR 7301 airfoil. Mader et al. [31] used TAPENADE and developed a discrete adjoint solver, which they called ADjoint. The resulting set of linear equations were assembled into a linear system, which was then solved using the preconditioned GMRES [39] routine of the PETSc [2] computational framework.

The difficulty in developing the adjoint form of the non-reflecting and the mixing plane boundary conditions has hindered its application for turbomachinery flows until recently [11,27,28,32,43–46]. Frey et al. [11] described a systematic approach to develop a discrete adjoint solver for turbomachinery optimization including the adjoint boundary conditions for the conservative mixing planes. Marta et al. [32] discussed physical meaning of steady adjoint solutions for turbomachinery. Luo et al. [28] presented optimization of the NASA Rotor 67 fan by using a three-dimensional viscous adjoint solver and redesigned the blades for three different operating conditions; namely, near peak efficiency, near stall, and near choke. Walther and Nadarajah [44] performed a constrained aerodynamic shape optimization for a transonic compressor in a multistage environment. In the literature to date, the turbomachinery design work using adjoint techniques were mostly limited to steady flows. A few notable exceptions that applied the technique to unsteady flows are the work of Campobasso et al. [4], Duta et al. [6] and He and Wang [18]. In two related works, Campobasso et al. [4], and Duta et al. [6] applied the adjoint method to a time-linearized turbomachinery flow solver and managed to reduce the forced response blade vibration by tailoring the shape of the incoming wakes, which can be related to the blade profile of the upstream row. More recently, He and Wang [18] performed an adjoint-based aerodynamic/aeroelastic design optimization of turbomachinery blades. Their method was based on a simplified nonlinear harmonic method and the optimization resulted in improved flutter characteristics.

In this work, we consider gradient-based shape optimization for improved aeroelastic properties of compressor cascades. Particularly, we optimize the blade shapes so as to maximize the aerodynamic damping of a vibrating cascade. Our technique, in some sense, is similar to the method of He and Wang except that we use the fully nonlinear time-spectral harmonic balance technique of Hall and that we use a discrete adjoint approach to determine the sensitivities for shape optimization. For two-dimensional inverse design problems, a new method is proposed where mesh points are used as design variables to provide a complete design basis and is shown to be very efficient in combination with a quasi-Newton optimization method. In addition, the unsteady adjoint technique is used to determine the frequency of vibration that would drive the system to limit-cycle oscillations (LCO), which defines the stability limit of the cascade. These studies constitute the novelty of this work. To obtain the discrete adjoint solver, we use the non-commercial automatic differentiation compiler TAPENADE. The adjoint solver is then coupled to an optimization algorithm for shape optimization or LCO frequency computation.

2. Methodology

2.1. Harmonic balance technique

The harmonic balance technique used herein has been extensively reported in earlier work (for example, see Refs. [16,20]). However, the salient features of the technique are repeated here for completeness. We consider two-dimensional Euler equations (the method can easily be extended to Navier–Stokes equations) written in strong conservation form as

$$\frac{\partial}{\partial t}\iint\limits_{\mathcal{A}} \mathbf{U}\,\mathrm{d}\mathcal{A} + \int\limits_{\mathcal{S}} [\mathbf{F},\mathbf{G}]\cdot\mathbf{n}\,\mathrm{d}\mathcal{S} = 0. \tag{1}$$

In the above equation, $\mathbf{U} = \{\rho, \rho u, \rho v, \rho E\}^T$ is the vector of conservation variables, and $\mathbf{F}$, $\mathbf{G}$ are the flux vectors given as

$$\mathbf{F} = \begin{Bmatrix} \rho u - \rho\dot{f} \\ \rho u^2 + p - \rho u\dot{f} \\ \rho u v - \rho v\dot{f} \\ \rho u h - \rho E\dot{f} \end{Bmatrix}, \qquad \mathbf{G} = \begin{Bmatrix} \rho v - \rho\dot{g} \\ \rho u v - \rho u\dot{g} \\ \rho v^2 + p - \rho v\dot{g} \\ \rho v h - \rho E\dot{g} \end{Bmatrix},$$

where $\dot{f}$ and $\dot{g}$ are the x and y components of the unsteady grid motion velocity.

Here, it is assumed that the cascade blades (and the computational grid) vibrate harmonically with a frequency ω. Because the flow is temporally periodic, the conservation variables may be approximated as a truncated Fourier series in time with spatially varying coefficients, which are computed and stored at $2N+1$ equally spaced points over one temporal period. Therefore, the Fourier coefficients and sub-time level solutions are related through a set of discrete transformation matrices, i.e.

$$\hat{\mathbf{U}} = \mathbf{E}\mathbf{U}^*; \qquad \mathbf{U}^* = \mathbf{E}^{-1}\hat{\mathbf{U}}, \tag{2}$$