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Geometrically nonlinear extended high order analysis for sandwich beams based on elastic–plastic core shear behavior

S. Jedari Salami, M. Sadighi [*], M. Shakeri

Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran

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ABSTRACT

Geometrically nonlinear response of a sandwich panel based on elastic–linear plastic (bilinear) shear behavior of the core is presented. The extended high order sandwich panel theory (EHSAPT) is applied to obtain governing equations. The nonlinear Von Karman type relations for strains of face sheets and the core are adopted. The face sheets follow first order shear deformation theory along with the linear elastic assumption. A Ritz based solution which is suitable for any essential boundary condition is implemented. Two types of boundary conditions are considered simply supported and immovable clamped at both edges. In each type of boundary conditions, the effects of plastic shear modulus of bilinear material behavior on transverse shear stress distribution along with detection of the elastic and plastic regions within the core, force and moment resultants, transverse displacement distributions in the face sheets, in-plane and vertical displacements through the thickness of the core are studied in detail. Besides, all results based on the bilinear material assumption of the core are compared with the linear elastic ones. Finally, for the case study of the simply supported beam, the results of the constitutive model are compared with finite element simulation.

1. Introduction

The application of sandwich structures composed of composite laminated face sheets and foam core has been extended widely in the last decades. The flexibility and lightness of the foam cores are the main reasons of their extensive usage in sandwich structures. The mechanical response of foam cores generally is described by a bilinear stress–strain curve based on experimental results. The typical curve includes three definite regions: linear elastic, plateau and densification. At small strains, the core is deformed in a linear elastic manner due to cell wall bending. As the load increases, the cells begin to collapse, depending on mechanical properties, by one of these cases: elastic buckling or plastic yielding. Collapse progresses at roughly constant load giving a stress plateau, until the opposing walls in the cells meet and touch when densification causes the stress to increase quickly [5]. These characteristics of the foam core have a significant effect on the behavior of the sandwich structure. Therefore, it should be noted that it is necessary to consider the bilinear behavior of the foam core for an accurate analysis of the behavior of the structure. For instance, the shear crushing of the core related with elastic–plastic behavior of the foam core occurs in marine applications because of impact or dynamic loadings. In these cases, the core undergoes large strain at closely uniform load in the plateau regime that may bring an untime failure [14]. There are several works to describe mechanical responses of different types of foam cores by bilinear stress–strain relations in the literature. Gibson and Ashby [5], Lubin [12] and Klintworth [9] characterized the behavior of a foam core and its stress–strain relation under out-of-plane and in-plane loadings. Zhu et al. [22] specified the nonlinear geometrical relations between the material type and its density on the form of the stress–strain curve. Sadighi and Jedari Salami [19] studied the mechanical behavior of elastomeric and crushable foams experimentally, and investigated the dependence of stress–strain curve on rate of loading. A literature survey on the nonlinear elastic–plastic behavior of the sandwich panel reveals there are very few works on this subject in the open literature. Mercado and Sikarskie [13] studied the response of sandwich panel based on a bilinear shear stress–strain relation. The classical sandwich theory assumptions where the height of the panel is kept fixed are considered in their work. To provide verification on the classical theory, a finite element (ABAQUS) solution was used as well. Jaskula and Zielnica [7] investigated stability analysis of elastic–plastic sandwich cylindrical shell for combination of loading. The J_2 incremental Prandtl–Reuss plastic flow theory was applied in this research. They assumed Kirchhoff–Love hypotheses considered for the shell. Therefore the core compression

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[*] Corresponding author. Tel.: +98 2164543448; fax: +98 2166419736.
E-mail address: mojtaba@aut.ac.ir (M. Sadighi).

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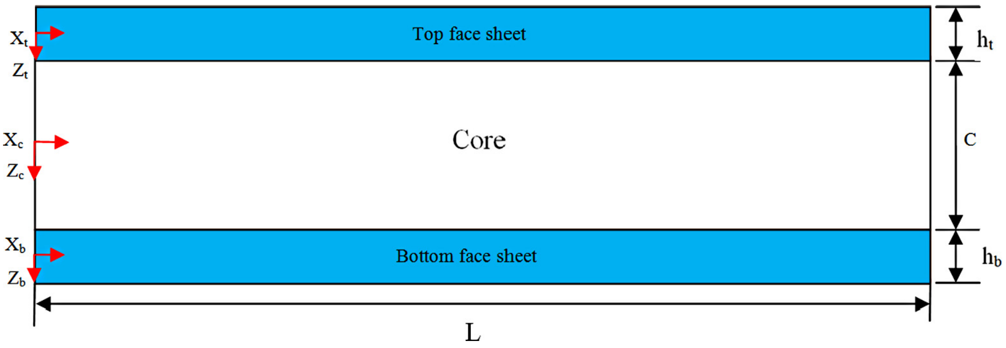

Fig. 1. Description of the geometrical configuration for the sandwich beam.

is supposed negligible. Liu et al. [11] have developed an analytical model to predict the elastic–plastic dynamic response of only fully backed sandwich panels under impulse load. The core is considered as an elastic–perfectly plastic foundation, where the top face sheet is supposed as a plate resting on the foundation. The resulted equations based on a minimum principle energy dealing with elastic–plastic analysis for the top face sheet are solved via the finite difference method. Li et al. [10] proposed an elastic–plastic model that employed the "bending hinge" concept to analyze the dynamic response of a simply supported sandwich beam. However, the core is assumed incompressible because of the classical beam theory assumptions are used in their work. Hoo Fatt and Chapagain [6] presented an analytical model for the damped, large amplitude response of a simply supported sandwich panel under pressure pulse loading. The foam core is considered elastic–perfect plastic material. The combined face sheets and core follow first order shear deformation plate theory; this in turn the core is assumed incompressible. Chang and Krauthammer [4] studied the elasto–perfectly plastic behavior of corrugated-core sandwich panel. The analysis is based on the incremental theory of plasticity combined with Hill's criterion. The field equations of the plate are derived based on the first order shear deformation plate theory. As seen, in all works, the elastic–plastic behavior of sandwich panels is analyzed within the framework of equivalent single-layer (ESL) theories. Therefore, transverse deformation of the core is neglected because of one displacement distribution at the sandwich mid-plane is considered to characterize deformation of the entire sandwich.

To date, there is no work reported on nonlinear structural behavior of a sandwich panel based on high order sandwich panel theory (HSAPT) except for study by Schwarts-Givli and Frostig [20]. The bending response of a sandwich panel with a transversely compressible bilinear core was investigated in that paper. The small deformation relations are used to obtain strains for all parts of sandwich structure. Also, the classic beam theory with negligible shear strain is used for the face sheets. The field equations as well as boundary and continuity conditions based on stress-displacement mixed variable model are derived via the variational principle of potential energy minimization.

In most of articles that have been used HSAPT, neglecting longitudinal stresses in core is a common assumption. It is because of low modulus of elasticity and low flexural rigidity of soft cores in comparison with those of the face sheets. Recently, Carlsson and Kardomateas [3] and Phan et al. [16] extended HSAPT for superior accuracy of results in all cases, particularly for the stiffer cores. This theory includes the in-plane rigidity of the core and can predict shear and in-plane stress distributions in the core. Also, in this approach three generalized coordinates in the core (in-plane and transverse displacement and rotation at centroid of the core) instead of just one (transverse displacement of the core) is considered. Phan et al. [15–17] used EHSAPT to investigate the bending, wrinkling and buckling of orthotropic sandwich beams and reported good accuracy of this approach. This approach assumes quadratic and cubic polynomial functions for the transverse and in-plane displacement fields of the core through the thickness, respectively. So, the unknowns in this model consist of displacements of the face sheets and coefficients of polynomials in the core.

As a progressive step, in this work, the geometrically nonlinear behavior of a sandwich panel based on an extended high order sandwich panel theory (EHSAPT) is presented. The elastic assumption is considered for the face sheets and the bilinear shear stress–strain behavior is adopted for the core. The strain–displacement relations of face sheets and the core follow the nonlinear Von Karman type. In this work, the face sheets and the core are considered Glare, the glass fiber–aluminum version of FMLs [21] and core-cell A-1200 foam, respectively. The main reason for considering fiber–metal laminate for face sheets is due to the ductile property of this type of material. Therefore, it is reasonable that the core may undergo plastic deformation while the face sheets remain elastic. In order to increase the accuracy of analysis, the face sheets follow first order shear deformation theory (FSDT). In this work, a Ritz based solution is appropriate to satisfy any applied essential boundary condition. Solution of the resulted nonlinear field equations is accomplished via the Newton–Raphson iterative method. Two types of boundary conditions are considered to be simply supported and immovable clamped at both edges. In each type of boundary conditions, the effects of bilinear material behavior for the shear stress–strain relation of the core on the transverse shear stress distribution along the panel are studied in detail. Also, the influences of the plastic shear modulus on the force and moment resultants, transverse displacement distributions in the face sheets, as well as the in-plane and vertical displacements through the thickness of the core, are investigated. Besides, all results based on the bilinear material assumption of the core are compared with the linear elastic ones.

2. Basic formulation

A sandwich beam of length (L) with a core of thickness (C), width of (b) and top and bottom face sheet thicknesses (h_t) and (h_b), respectively is shown in Fig. 1. As it is mentioned above, a sandwich beam is formulated within the framework of extended high order sandwich panel theory (EHSAPT). The assumptions for the analysis are considered as follows:

1) The face sheets are treated as a linear elastic material and formulated based on first order shear deformation theory (FSDT).
2) The isotropic core is assumed as a 2D elastic medium where transverse flexibility and transverse normal stress and strain are considered.