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Nonlinear topology optimization of centrifugally loaded aero-engine part with newly developed optimality-criteria based algorithm

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ABSTRACT

In this paper a successful topology optimization of a centrifugally loaded aero-engine part is presented. For the topology optimization, which is a nonlinear problem due to several contact regions, a self-developed topology optimization algorithm in combination with a commercial FE-solver is used. The goal of the optimization was the reduction of stresses in the structure.

1. Introduction

Since many years the topology optimization is a well-established and widely used tool in structural mechanics. However in most cases only linear problems are investigated although there are some approaches for nonlinear topology optimization algorithms like the equivalent static loads method [7] for sensitivity-based algorithms or algorithms basing on optimality criteria like ESO-methods (**E**volutionary **S**tructural **O**ptimization, for details see [12]) or hybrid cellular automata methods [8]. Here, we will present a very simple and easy to implement topology optimization algorithm, which reached very good results in some linear and nonlinear test-problems.

In the second part of this paper the introduced algorithm is used for the topology optimization of an aero-engine part, which is loaded by very high centrifugal loads. The topology optimization of such body-force problems is a difficult task and can cause failure of conventional sensitivity-based topology optimization algorithms [2].

Because the Finite Element Method (FEM) is the standard approach to solve boundary value problems of arbitrary structures for many decades, all further explanations and comments have to be considered against this background.

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2. New topology optimization algorithm

The presented topology optimization algorithm is an optimality criteria-based method. The fundamental idea behind these methods is the so-called fully-stressed design. This means that a design is considered as optimal, if all regions of the structure or all infinitely small volume elements of a continuum resp. are subjected to the same maximum allowable stress state.

In order to reach such a fully-stressed design, different approaches are possible. The algorithm used here, iteratively increases the stiffness of finite elements with high stresses and decreases the stiffness of low-stressed elements. For the introduction of design variables in the optimization process the SIMP-approach (**S**olid **I**sotropic **M**aterial with **P**enalization, see also [1]) is used. In this approach the stiffness tensor $\boldsymbol{E}$ of a finite element is computed as the product of the "normal" stiffness tensor $\boldsymbol{E}_0$ of the element, which depends on geometrical and material properties of the element, and a dimensionless so-called density factor ρ:

$$\boldsymbol{E} = \rho^p \boldsymbol{E}_0. \tag{1}$$

The density factor ρ may vary between 0 and 1 where $\rho > 0$ should be always true to avoid numerical problems. The exponent $p \geq 1$ is called penalty exponent. It is introduced to force the optimization algorithm to come up with a clear 0–1 distribution of material. If we assume for example, that an element has a density factor of $\rho = 0.5$ and p equals 2, then the volume of the element is computed as $V = 0.5 \cdot V_0$ (with the geometrical element volume V_0) but the stiffness follows as $\boldsymbol{E} = 0.5^2 \boldsymbol{E}_0 = 0.25 \boldsymbol{E}_0$. This

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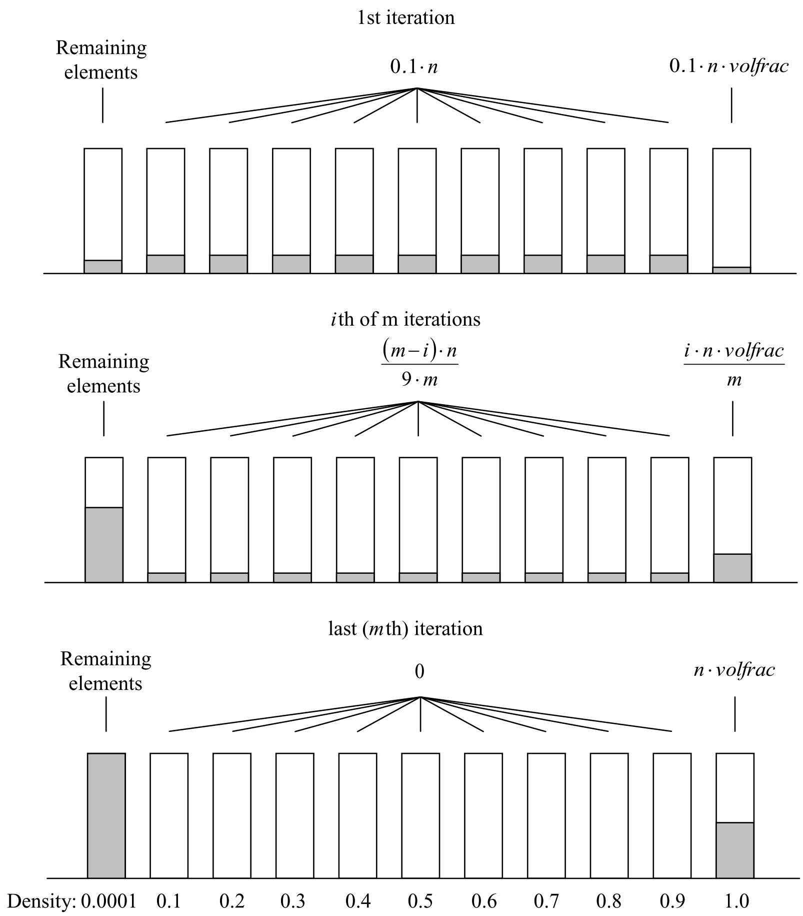

Fig. 1. Change of density distribution during the optimization process (at the example of a volume constraint of 30%) [6].

means that the element causes high costs with respect to the volume but offers a low stiffness compared to these costs. Now the optimization algorithm must decide if the element is important for the structure and increase the stiffness or decrease the stiffness if the element is not important. Both would improve the cost–benefit ratio. The advantage of penalty exponents greater than 1 is that the result becomes more discrete and contains fewer elements with "intermediate densities", which makes an interpretation of the result much easier. The disadvantage is that optimization problems, which are actually convex, lose their convexity [11,10]. After all, the task for the optimization algorithm is to determine an optimal density factor for every finite element. At the end of the optimization, the optimized density distribution shows, which regions of the meshed design space are important for the structure and their desired properties (high density factor and therefore high stiffness) and which regions are not important due to their low density and therefore low stiffness.

The algorithm presented here is able to solve topology optimization problems of the form

$$\min_{\rho} \boldsymbol{f}^T \boldsymbol{u} \quad s.t. \quad \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{u} = \boldsymbol{f}, \quad \frac{\boldsymbol{\rho}^T \boldsymbol{v}}{V_{ds}} = volfrac, \quad 0 < \rho_i \le 1, \quad (2)$$

where $\boldsymbol{f}$ is the global load vector, $\boldsymbol{u}$ is the global displacement vector, $\boldsymbol{K}$ is the global stiffness matrix (which depends on the vector of the element densities $\boldsymbol{\rho} = (\rho_1, \rho_2, ..., \rho_n)^T$), $\boldsymbol{v}$ is the vector of the geometrical element volumes, V_{ds} is the sum of all geometrical element volumes V_i in the design space, *volfrac* is a volume constraint, which defines the ratio of the desired final volume of the design space to the total (geometrical) design space volume V_{ds} and n is the number of finite elements in the design space. This means the algorithm is able to solve a so-called minimum compliance problem (maximization of global stiffness under volume constraint). The optimal density distribution is determined by using a "container" system, which is also explained in Fig. 1. Every container represents one of the eleven density values $0.0001, 0.2, \ldots, 0.9$ or 1.0. All elements are iteratively assigned to one of the eleven containers and get the density value corresponding to the respective container. (The penalty exponent p equals always 1 in this algorithm.) For the classification of the elements to the containers, the von Mises stress of the elements is evaluated (if there are several integration points in one element, the average value is computed) in the examples, presented in this paper. In general not only the von Mises equivalent stress has to be used but all equivalent stress formulations, that are proportional to the strain energy density, are usable [1,9]. After each iteration, the density distribution is determined again. Elements with a high von Mises stress are classified to a container with a high density value and elements with a low von Mises stress level are classified to a container with a low density. With an increasing number of iterations, the number of elements with densities between 0 and 1 (e.g. 0.5 or 0.6) is reduced and the design is driven to a clear 0–1 or white–black distribution respectively. This simplifies the interpretation of the result at the end of the optimization. Finally, there are only a few elements with intermediate densities left to fulfill the volume constraint of the optimization task.

In the first iteration $i = 1$, the $0.1 \cdot n \cdot volfrac$ elements with the highest von Mises stress are assigned to the container with the density 1.0. Consecutively the containers with the densities $0.1, \ldots, 0.9$ are each filled with $0.1 \cdot n$ elements. If two elements have exactly the same von Mises stress value, they are assigned to the same container even if the maximum number of elements for the container is already reached. This procedure ensures the