Aerospace Science and Technology 38 (2014) 48–55

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Inverse analysis for transient thermal load identification and application to aerodynamic heating on atmospheric reentry capsule

Toshiya Nakamura [a,*], Yukihiro Kamimura [b], Hirotaka Igawa [a], Yoshiki Morino [b]

[a] Structures Research Group, Japan Aerospace Exploration Agency, 6-13-1 Osawa, Mitaka, Tokyo 181-0015, Japan
[b] Dept. Applied Mechanics and Aerospace Engineering, Waseda University, 3-4-1 Okubo, Shinjuku, Tokyo 169-8555, Japan

ARTICLE INFO

Article history:
Received 24 December 2013
Received in revised form 30 July 2014
Accepted 31 July 2014
Available online 11 August 2014

Keywords:
Transient inverse heat conduction analysis
Pseudo-inverse matrix
Sequential function specification
Truncated singular value decomposition
Reentry capsule

ABSTRACT

A transient inverse heat conduction analysis enables the identification of the unknown boundary heat flux from finite number of temperature data obtained, e.g., in the high temperature structural tests or in the actual operation. Since the prediction of thermal load (heat flux) is difficult, the inverse analysis is expected to improve structural design of high temperature components. The present study develops a computational method of transient inverse heat conduction analysis. The developed code is applied to problems of a simple two-dimensional plate and of an atmospheric reentry capsule. Sequential Function Specification (SFS) method and Truncated Singular Value Decomposition (TSVD) are employed to improve the stability of the inverse analysis. Effects of these regularization methods are numerically discussed.

1. Introduction

Thermal analysis plays an important role in the structural design of high temperature components. However, prediction of thermal load (heat flux) is difficult due to physical complexity and uncertainties. The effect of uncertainties is significant in some cases and the probabilistic method is one of the promising approaches in the structural analysis [17]. One of the authors investigated the transient probabilistic thermal responses of a reentry capsule structure and its thermal reliability was assessed by Monte Carlo simulation [13].

Consequently it is useful if actual or operational thermal loads are identified from the temperature data obtained, e.g., in the high temperature structural tests, high enthalpy wind tunnel tests or actual reentry flights. These identified load data may contribute to improving the load prediction method and hence the structural design [1], and to understanding the phenomena as well. As to the mechanical load identification, so-called operational load monitoring has a long history in many engineering fields including aerospace, but it is still attractive due to the significant progress of sensors and data processing technologies today. The authors' group proposed a flexible method to reconstruct continuously distributed load (pressure) from finite number of strain data based on inverse elastic analysis [14].

The transient inverse heat conduction analysis is a way to estimate heat flux distribution and its history from finite number of temperature data. It has been studied by some researchers [3–6, 15]. However, many of them are based on the finite difference method and can only be used for problems of simple geometry. Duda et al. developed a space marching method, which can be used to solve an inverse multidimensional heat conduction problem with complex-shaped bodies [5].

Generally, the transient inverse analysis is sensitive to the measurement error, measurement locations, and computational time step, but there have been few literatures on this issue. Ling and Atluri presented a method to analyze the error propagations in this problem [12].

In the present study, a computational method of transient inverse heat conduction analysis is developed to estimate the heat load from the temperature data. Our approach is based on the study by Duda [5]. In addition, a discrete representation of the boundary condition, which was proposed by the authors [14], is incorporated to allow for an approximation of the continuous heat load distribution with small number of degree of freedom (DOF). Applications to the simple two-dimensional problem and to the atmospheric reentry flight are demonstrated. The time-dependent heat flux distributions are reconstructed form the measured temperature data. The discussion will be focused on the computational stability and the regularization. The effects of the Sequential Function Specification (SFS) and the Truncated Singular Value Decomposition (TSVD) will be numerically investigated.

* Corresponding author. Tel.: +81 50 3362 7014; fax: +81 422 40 1434.
E-mail address: nakamt@chofu.jaxa.jp (T. Nakamura).

http://dx.doi.org/10.1016/j.ast.2014.07.015
1270-9638/

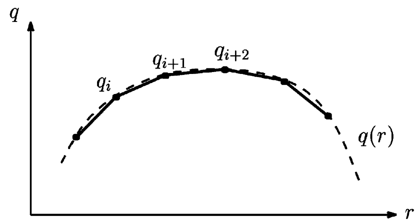

Fig. 1. Discrete modeling of boundary condition.

It should be noted that another background of this study is the recent development of optical sensors. One of the authors has developed a long gauge FBG (Fiber Bragg Grating) sensor using OFDR (Optical Frequency Domain Reflectometry) technique. OFDR has a superior potential capable of measuring strains and temperatures at an arbitrary position along the fiber [10,11]. The indication is that thousands of strain and temperature data will be acquired with less efforts, leading us to develop a load identification technique based on inverse analysis, because the inverse analysis requires a large amount of data to obtain a stable solution.

2. Procedure of inverse analysis

2.1. Fundamental equations

The transient heat conduction equation within a body Ω is

$$\rho c \dot{T}(\boldsymbol{r}, t) = \nabla \cdot \big(\kappa \nabla T(\boldsymbol{r}, t)\big), \quad \boldsymbol{r} \in \Omega, \tag{1}$$

where material properties c (specific heat), ρ (density), and κ (thermal conductivity) are assumed to be independent of temperature for simplicity, and the heat generation within the body is out of consideration.

We have an initial condition as

$$T(\boldsymbol{r}, t_0) = T_0, \quad \boldsymbol{r} \in \Omega. \tag{2}$$

The boundary conditions on the boundary $\partial\Omega = \partial\Omega_T \cup \partial\Omega_q \cup \partial\Omega_h$ are as follows.

$$\left.\begin{array}{lll} \text{Specified temperature:} & T(\boldsymbol{r}, t) = \bar{T}(\boldsymbol{r}, t), & \boldsymbol{r} \in \partial\Omega_T \\ \text{Specified heat flux:} & \kappa \dfrac{\partial T}{\partial n}(\boldsymbol{r}, t) = q(\boldsymbol{r}, t), & \boldsymbol{r} \in \partial\Omega_q \\ \text{Convection:} & \kappa \dfrac{\partial T}{\partial n}(\boldsymbol{r}, t) = h(T_\infty - T_{\Omega_h}), & \boldsymbol{r} \in \partial\Omega_h \end{array}\right\} \tag{3}$$

where n in Eq. (3) stands for the normal to the corresponding boundary. $\bar{T}$, q, T_∞, T_{Ω_h} are the specified temperature, heat flux, environmental temperature, and structural temperature on $\partial\Omega_h$, respectively, and h is the convection coefficient.

The inverse problem is to identify the boundary condition as a function of $\boldsymbol{r} \in \partial\Omega$ and time from finite number of temperature data. The present study considers only the heat flux (Eq. $(3)_2$) for the boundary condition.

2.2. Discretization of boundary condition

In the authors' previous study of the inverse elastic analysis [14], the continuously distributed mechanical load (pressure distribution) was approximately represented by the combination of shape functions and nodal values. The unknown nodal values were identified from the finite number of strain data by the inverse operation. In the present study, we use the same method for the thermal boundary condition.

As shown in Fig. 1, the spatially continuous heat flux distribution is approximated by Eq. (4).

$$q(\boldsymbol{r}, t) = \sum_{i=1}^{N_q} N_i(\boldsymbol{r}) q_i(t) \tag{4}$$

where q is the boundary heat flux and q_i is that at the i-th node. $N_i(\boldsymbol{r})$'s are the linear shape (interpolation) functions, and N_q is the number of nodes for the boundary condition approximation. The advantage of this method is that the DOF of the inverse analysis N_q can be determined independently of the finite element model so that the stable solution is obtained.

2.3. Incremental form

Ordinary finite element (FE) formulation yields the spatially discretized form of Eq. (1).

$$\boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{R} \tag{5}$$

where $\boldsymbol{x}$ is a vector of nodal temperature of the FE model with the total number of nodes of N_x, that is,

$$\boldsymbol{x}(t) = \big(T_1(t), T_2(t), \cdots, T_{N_x}(t)\big)^T. \tag{6}$$

$\boldsymbol{C}$ and $\boldsymbol{K}$ are the capacitance matrix and the conductance matrix, respectively, and $\boldsymbol{R}$ is the thermal load vector [16].

We rewrite Eq. (5) in the form of Eq. (7) [5].

$$\dot{\boldsymbol{x}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{q} \tag{7}$$

where $\boldsymbol{q}$ is a vector of the nodal values of the boundary heat flux in the form of Eq. (4), thus,

$$\boldsymbol{q}(t) = \big(q_1(t), q_2(t), \cdots, q_{N_q}(t)\big)^T. \tag{8}$$

The initial condition is

$$\boldsymbol{x}(t_0) = \big(T_1(t_0), T_2(t_0), \cdots, T_{N_x}(t_0)\big)^T. \tag{9}$$

The dimensions of the coefficient matrices $\boldsymbol{F}$ and $\boldsymbol{G}$ are $N_x \times N_x$ and $N_x \times N_q$, respectively.

The incremental form of Eq. (7) can be written as [5]

$$\boldsymbol{x}\big((k+1)\Delta t\big) = \boldsymbol{A}\boldsymbol{x}(k\Delta t) + \boldsymbol{B}\boldsymbol{q}(k\Delta t) \tag{10}$$

where $N_x \times N_x$ matrix $\boldsymbol{A}$ and $N_x \times N_q$ matrix $\boldsymbol{B}$ depend on Δt.[1]

Using the time step k for the time t, we have the following incremental form.

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{q}(k) \\ \boldsymbol{y}(k) = \boldsymbol{M}\boldsymbol{x}(k) \end{cases} \tag{11}$$

where $\boldsymbol{y}(k)$ is a vector of temperature data at time step k, whose elements are

$$\boldsymbol{y}(k) = \big(T_1^m(k), T_2^m(k), \cdots, T_{N_y}^m(k)\big)^T. \tag{12}$$

$N_y \times N_x$ matrix $\boldsymbol{M}$ extracts the temperature data at the measurement locations from $\boldsymbol{x}$ (the full set of temperature at the entire nodes of FE model).

[1] Mathematical representations of the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are as follows [5].

$$\boldsymbol{A} = \boldsymbol{I} + \sum_{n=1}^{\infty} \frac{1}{n!}(\boldsymbol{F}\Delta t)^n, \quad \boldsymbol{B} = \left\{\boldsymbol{I} + \sum_{n=1}^{\infty} \frac{1}{(n+1)!}(\boldsymbol{F}\Delta t)^n\right\}\boldsymbol{G}\Delta t$$

We determined these coefficient matrices numerically through the ordinary finite element formulation with fixed Δt. Eq. (10) is integrated for Δt (one step) by FEM with $\boldsymbol{x}(0) = (1, 0, \cdots, 0)^T$ and $\boldsymbol{q}(0) = \boldsymbol{0}$. The result is $\boldsymbol{x}(\Delta t) = (A_{11}, A_{12}, \cdots, A_{1N_x})^T$. This process is repeated to determine all components of $\boldsymbol{A}$ and $\boldsymbol{B}$.