Aerospace Science and Technology 38 (2014) 64–75

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Formulation of a new set of Simplified Conventional Burnett equations for computation of rarefied hypersonic flows

Wenwen Zhao [a], Weifang Chen [a], Ramesh K. Agarwal [b,*,1]

[a] Zhejiang University, Hangzhou 310027, China
[b] Washington University in St. Louis, Saint Louis 63130, USA

ARTICLE INFO

Article history:
Received 3 March 2014
Received in revised form 8 July 2014
Accepted 29 July 2014
Available online 5 August 2014

Keywords:
Burnett equations
Hypersonic
Rarefied gas dynamics
Transitional flow

ABSTRACT

For computation of rarefied flows in continuum-transition regime with Knudsen number *Kn* of O(1), Burnett equations have been proposed about a century ago as a set of extended hydrodynamics equations (EHE) that represent the second-order departure from thermodynamic equilibrium in the Chapman–Enskog expansion of Boltzmann equation; the first order terms in the expansion result in the Navier–Stokes equations. Over the years, a number of variations of original Burnett equations have been proposed in the literature known as the Conventional Burnett equations, the Augmented Burnett equations and the BGK–Burnett equations. In this paper, another simpler set of Burnett equations is proposed by order of magnitude analysis in the limit of high Mach numbers for hypersonic flow applications. These equations, designated as 'Simplified Conventional Burnett (SCB)' equations are stable under small perturbations and do not violate the second law of thermodynamics. An implicit numerical solver is developed for the solution of SCB equations. The SCB equations are applied to compute the hypersonic flow past 2D and 3D blunt bodies for *Kn* in continuum and continuum-transition regime. The SCB solutions are compared with the Navier–Stokes and DSMC solutions. It is shown that the SCB equations can be employed to compute the hypersonic flow past bodies in continuum-transition regime with much less computational effort because of their simplicity compared to Conventional and Augmented Burnett equations.

1. Introduction

In high altitude hypersonic flows past space vehicles, especially in low earth orbit, part of the flow field is in continuum regime and part of it is in rarefied continuum-transition regime where Knudsen number $Kn \sim O(1)$. This is also the case for gaseous flow in many micro-electro-mechanical (MEM) devices where the low density and small length scale result in $Kn \sim O(1)$. In continuum-transition regime, Navier–Stokes (NS) equations are not accurate because of continuum breakdown due to rarefaction in flow regions near the bow shock, in Knudsen layer and in the wake of the vehicle [8]. Direct Simulation Monte Carlo (DSMC) [5,6] has been widely used for calculation of rarefied flows; however it requires a large number of particles which make the simulations computationally very costly. As an alternative, higher-order extended hydrodynamics equations (EHE) beyond Navier–Stokes have been proposed for computation of flows in continuum-transition regime. These equations are derived from the classical Boltzmann equation by applying the Chapman–Enskog expansion; they were first derived by Burnett and therefore known as the Burnett equations [9]. They represent the second-order departure from thermodynamic equilibrium; the first-order departure leads to the Navier–Stokes equations. These equations include higher-order stress tensor and heat flux terms in the constitutive relations. It is instructive to recall that the constitutive relations for Navier–Stokes equations are linear Stokes law for the stress tensor and Fourier's law for the heat flux. Over the years, a number of variants to the original Burnett equations [9] have been proposed; these are summarized in Refs. [1,13] and are known as the 'Conventional Burnett equations (CBE)', 'Augmented Burnett (AB) equations' and the 'BGK–Burnett equations'.

One of the difficulties in using Burnett equations has been that they have been found to be unstable under small wavelength perturbations; this was concluded by Bobylev [7] by conducting the linearized stability analysis of 1D Burnett equations. This problem of stability was also noted by Fiscko and Chapman [12] and Fiscko [11] who found that the Burnett equations became unstable when the mesh was refined. For a long time, the linearized instability problem impeded the use of Burnett equations

* Corresponding author. Tel.: +1 314 935 6091.
E-mail address: agarwalr@seas.wustl.edu (R.K. Agarwal).
1 William Palm Professor in Department of Mechanical Engineering & Materials Science.

http://dx.doi.org/10.1016/j.ast.2014.07.014
1270-9638/

Nomenclature

a	sound velocity ... m/s	T_w	wall temperature ... K
r	nose radius ... m	T_s	jump-temperature ... K
k	non-dimensional circular frequency	U	free stream velocity ... m/s
K_i	coefficients of stress terms in Burnett equations	u_i	velocity tensor ... m/s
Kn	Knudsen number	u_s	slip-velocity ... m/s
L	characteristic length ... m	ρ	density ... kg/m^3
L_{ref}	reference length ... m	ρ_∞	free stream density ... kg/m^3
Ma	Mach number	ρ_{ref}	reference density ... kg/m^3
p	pressure ... N/m^2	τ_{ij}	stress tensor ... N/m^2
p_∞	free stream pressure ... N/m^2	μ	viscosity ... (N·s)/m^2
Pr	Prandtl number	μ_{ref}	reference viscosity ... (N s)/m^2
q_i	heat flux terms ... J/(m^2 s)	κ	thermal conductivity ... J/(m s K)
R	gas constant ... J/(kg K)	θ_i	coefficients of heat flux terms in Burnett equations
Re	Reynolds number	λ	mean free path of gas molecules ... m
T	temperature ... K	σ_u	accommodation coefficient of momentum
T_∞	free stream temperature ... K	σ_T	accommodation coefficient of temperature
T_{ref}	reference temperature ... K		

for computation of rarefied flows. In 1993, Zhong et al. [25] derived the third-order super Burnett equations and then introduced part of third-order terms into the conventional Burnett equations to stabilize them using the Bobylev's method; they called this new set of equations the Augmented Burnett (AB) equations. However, the addition of third-order terms added to the complexity and in some cases it has been difficult to obtain convergent stable solutions [1]. Comeaux et al. [10] and Jin and Slemrod [14] studied the compatibility of the conventional Burnett equations with the second law of thermodynamics and concluded that the addition of some third-order terms can result in violation of this law. Balakrishnan et al. [3] and Balakrishnan and Agarwal [2] derived a new set of Burnett equations by using the Bhatnagar–Gross–Krook (BGK) model for the collision integral in the classical Boltzmann equation; they called this new set as the BGK–Burnett equations. Balakrishnan and Agarwal [2] showed that the BGK–Burnett equations are stable under small wave length disturbances and satisfy the Boltzmann H-theorem (they do not violate the second law of thermodynamics). It is also important to note the observation of Welder et al. [18] that it is not sufficient to carry out the linearized stability analysis by ignoring the high-order non-linear terms in Burnett equations to ensure stable solutions especially at larger Knudsen numbers; it does not guarantee the stability of non-linear equations on fine computational grids.

Because of their complexity, there has been limited development of 3D computational codes in generalized coordinate system for the solution of Burnett equations for rarefied flows past complex geometries. Majority of the papers consider computation of one-dimensional shock structure using Burnett equations. Among 2D and 3D applications, Zhong et al. [22,24] studied the hypersonic flow past a 2D cylindrical leading edge and axisymmetric flow past blunt bodies using the AB equations while Yun [20] investigated more complex 3D cases in his Ph.D. dissertation using the AB equations. Yun and Agarwal [21] also compared the rarefied hypersonic flow simulations using the BGK Burnett equations and the Augmented Burnett equations. Recently, Bao et al. [4] have performed the 2D linearized stability analysis of different variants of Burnett equations using the Bobylev's method and have shed additional light on the stability issue in higher dimensions.

In this paper we derive a new set of Burnett equations which are simpler than the conventional Burnett equations by performing an order of magnitude analysis of higher-order stress and heat-flux terms and neglecting the terms which become negligibly small in the limit of large Mach number. We call this new set as the Simplified Conventional Burnett (SCB) equations, which is unconditionally stable according to one-dimensional linearized stability analysis. These equations are then solved by an implicit LU symmetric Gauss–Seidel scheme. The Developed SCB code is applied to calculate 2D and 3D blunt body flows and the results are compared with NS solutions and Augmented Burnett solutions as well the DSMC solutions. For hypersonic flow past a 2D cylinder, it is shown that the SCB equations are computationally 33% more efficient than the AB equations and 25% less efficient than the NS equations.

2. Derivation of a new set of Simplified Conventional Burnett (SCB) equations

The original Burnett equations were derived from the classical Boltzmann equation for monoatomic gases employing the Chapman–Enskog expansion to the velocity distribution function to second order in Knudsen number, that is to O(Kn^2) [9]. The conventional Burnett equations [1] were derived by replacing all the material derivatives in the original Burnett equations by the expressions for the material derivatives in the Euler equations. In the conventional Burnett equations, the second-order terms for the stress tensor and heat flux can be written as:

$$\begin{aligned}\tau_{ij}^{(2)} &= K_1\frac{\mu^2}{p}\frac{\partial u_k}{\partial x_k}\overline{\frac{\partial u_i}{\partial x_j}} \\ &\quad + K_2\frac{\mu^2}{p}\left(-\overline{\frac{\partial}{\partial x_i}\frac{1}{\rho}\frac{\partial p}{\partial x_j}} - \overline{\frac{\partial u_k}{\partial x_i}\frac{\partial u_j}{\partial x_k}} - 2\overline{\overline{\frac{\partial u_i}{\partial x_k}}\frac{\partial u_k}{\partial x_j}}\right) \\ &\quad + K_3\frac{\mu^2}{\rho T}\overline{\frac{\partial^2 T}{\partial x_i\partial x_j}} + K_4\frac{\mu^2}{\rho^2 RT^2}\overline{\frac{\partial p}{\partial x_i}\frac{\partial T}{\partial x_j}} \\ &\quad + K_5\frac{\mu^2}{\rho T^2}\overline{\frac{\partial T}{\partial x_i}\frac{\partial T}{\partial x_j}} + K_6\frac{\mu^2}{p}\overline{\overline{\frac{\partial u_i}{\partial x_k}}\frac{\partial u_k}{\partial x_j}}\end{aligned} \tag{1}$$

$$\begin{aligned}q_i^{(2)} &= \theta_1\frac{\mu^2}{\rho T}\frac{\partial u_k}{\partial x_k}\frac{\partial T}{\partial x_i} + \theta_2\frac{\mu^2}{\rho T}\left[\frac{2}{3}\frac{\partial}{\partial x_i}\left(T\frac{\partial u_k}{\partial x_k}\right) + 2\frac{\partial u_k}{\partial x_i}\frac{\partial T}{\partial x_k}\right] \\ &\quad + \left(\theta_3\frac{\mu^2}{\rho p}\frac{\partial p}{\partial x_k} + \theta_4\frac{\mu^2}{\rho}\frac{\partial}{\partial x_k} + \theta_5\frac{\mu^2}{\rho T}\frac{\partial T}{\partial x_k}\right)\overline{\frac{\partial u_k}{\partial x_i}}\end{aligned} \tag{2}$$

where a bar over the derivative in Eqs. (1) and (2) represents a non-divergent symmetrical tensor defined below:

$$\overline{f_{ij}} = \frac{1}{2}(f_{ij} + f_{ji}) - \frac{1}{3}\delta_{ij}f_{kk} \tag{3}$$