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Almost global asymptotic tracking control for spacecraft body-fixed hovering over an asteroid

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ABSTRACT

Almost global asymptotic tracking control for autonomous body-fixed hovering of a rigid spacecraft over an asteroid is proposed in the framework of geometric mechanics. The configuration space for the spacecraft is the Lie group SE(3), which is the set of positions and orientations of the rigid spacecraft in three-dimensional Euclidean space. The relative motion with respect to the spacecraft is assumed to be available through the spacecraft on-board navigation. The spacecraft tracks a desired relative configuration with respect to an asteroid in an autonomous manner. The relative configuration between the spacecraft and the asteroid is described in terms of exponential coordinates on the Lie group of rigid body motions. A continuous-time feedback tracking control using these exponential coordinates and the relative velocities is presented to perform coupled translational and rotational maneuver over an asteroid in the presence of control force saturation. A Lyapunov analysis guarantees that the spacecraft asymptotically converges to the desired trajectory. Numerical simulation results demonstrate the asymptotic tracking control achieve autonomous body-fixed hovering over a selected asteroid.

1. Introduction

Spacecraft hovering around small bodies such as asteroids and comets is essential for performing scientific explorations. In the hovering maneuver, the spacecraft may need to use control thrust continuously, overly continuously, or intermittently to null out gravitational and rotational accelerations, creating an equilibrium at a desired position in oder to maintain the desired position [25,28]. This approach is feasible over small bodies because the nominal accelerations on a spacecraft are small [5,6,27]. In general, there are two types of approaches in spacecraft hovering around an asteroid: over-inertial hovering and body-fixed hovering [5,27,28]. In over-inertial hovering the spacecraft fixes its position relative to the asteroid in the rotating asteroid–Sun frame, creating an artificial libration point in this frame. This hovering mode was implemented by the Hayabusa during most of its mission [28]. As the counterpart to inertial or over-inertial hovering, in asteroid body-fixed hovering the spacecraft fixes its position relative to the rotating asteroid and rotates with the asteroid in inertial space. In addition, every maneuver is performed relative to the asteroid body-fixed frame. This hovering mode was also implemented by the Hayabusa mission [10,12,13] during its sampling performances over the surface [5,28]. Body-fixed hovering is necessary for the spacecraft to sample a small body surface by controlling its motion in the asteroid body-fixed frame. Hovering trajectories can be implemented for many rotation periods (hours) of the asteroid with a modest control thrust, since the gravitational attraction is relatively weak at the asteroid. In general, the asteroid rotation period is on the order of hours to days at most [28]. Additionally, the spacecraft typically must reorient its attitude to maintain the same attitude relative to the asteroid body-fixed frame for the hovering maneuver time.

Most recent studies on spacecraft hovering about asteroids have focused mainly on achieving the desired relative position of a point-mass spacecraft with respect to the asteroid without performing attitude maneuvers to maintain the desired attitude [2,5,6,9,13,25,27]. However, this paper presents a rigid body spacecraft hovering scheme for a uniformly rotating asteroid whose rotation rate is constant in the asteroid body-fixed frame [11,28,29] where the spacecraft is required to implement maneuvers with large ranges of rotational motion in three-dimensional Euclidean

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Nomenclature

SE(3)	Lie group or set of positions and orientations of the rigid spacecraft moving in three-dimensional Euclidean space	
SO(3)	Lie group of orientations of the rigid body	
$\mathfrak{so}(3)$	Lie algebra of SO(3), which is represented as the nonlinear space of 3×3 skew-symmetric matrices	
$\mathfrak{se}(3)$	Semi-direct product of $\mathbb{R}^3$ and $\mathfrak{so}(3)$ and isomorphic to the vector space $\mathbb{R}^6$	
$\mathbb{R}^6$	Six dimensional real Euclidean vector space	
$\mathbb{R}^3$	Three-dimensional real Euclidean space of positions of the center of mass of the body	
b	Inertial position vector	m
ν	Translational velocity in the body-fixed frame	m/s
Ω	Angular velocity in the body-fixed frame	rad/s
F_g	Gravity force in the body-fixed frame	N
M_g	Gravity-gradient moment in the body-fixed frame	Nm
μ_S	Gravitational parameter of the Sun	$\mathrm{m}^3/\mathrm{s}^2$
μ_a	Gravitational parameter of the asteroid	$\mathrm{m}^3/\mathrm{s}^2$
μ_c	Gravitational parameter of the spacecraft	$\mathrm{m}^3/\mathrm{s}^2$
g	Configuration of the spacecraft on SE(3)	
ξ	Vector of body velocities of the spacecraft	rad/s, m/s
Ad	Adjoint actions of $g \in \mathrm{SE}(3)$ on $X \in \mathfrak{se}(3)$	
ad	Adjoint representation of $\mathfrak{se}(3)$	
$\mathrm{ad}^{\star}$	Co-adjoint representation of $\mathfrak{se}(3)$	
φ_g^0	Vector of known gravity inputs (moment and force) on the asteroid due to the Sun	Nm, N
φ_{G_S}	Vector of known gravity inputs (moment and force) on the spacecraft due to the Sun	Nm, N
φ_{G_a}	Vector of known gravity inputs (moment and force) on the spacecraft due to the asteroid	Nm, N
φ_c	Vector of control inputs (torque and force) of the spacecraft	Nm, N
h_f	Fixed relative configuration of the spacecraft to the virtual leader	
h	Relative configuration of the spacecraft to the virtual leader	
$\tilde{\eta}$	Exponential coordinate vector for the configuration error of the spacecraft	rad, m
expm	Exponential map from $\mathfrak{se}(3)$ to SE(3)	
logm	Logarithm map SE(3) to $\mathfrak{se}(3)$ (inverse of the exponential map)	
$\tilde{\Theta}$	Exponential coordinate vector for the attitude tracking error (principal rotation vector) of the spacecraft	rad
$\tilde{\beta}$	Exponential coordinate vector for the position tracking error of the spacecraft	m
$\tilde{\xi}$	Relative velocities of spacecraft with respect to the asteroid	rad/s, m/s
C_{20}, C_{22}	Second degree and order gravity field coefficients	km^2

space while tracking a desired trajectory. Thus, the configuration space for the spacecraft modeled as a rigid body is the Lie group SE(3) [3,7,17,18,22–24,31], which is the set of positions and orientations of the spacecraft moving in three-dimensional Euclidean space. General treatments of mechanical systems whose configuration spaces are Lie groups are given in Refs. [3,18]. The body-fixed spacecraft hovering is carried out with reference to the asteroid. The asteroid's trajectory is assumed to be obtained from the known dynamics model of a rigid body in a central gravity field. This trajectory can be computed off-line and is known to the spacecraft in order to implement the body-fixed hovering.

For this goal, a continuous-time feedback trajectory tracking control scheme [15,16] for three dimensional translational and rotational maneuvers over an asteroid, that ensure autonomous trajectory correction [1] to the desired trajectory is proposed. This scheme is shown to asymptotically track the desired configuration, using a Lyapunov analysis. The desired trajectory for the spacecraft is obtained by having a constant desired relative configuration between the spacecraft trajectory and the asteroid trajectory. The relative configuration (pose) of the spacecraft with respect to the asteroid is represented by exponential coordinates on SE(3). Thus, the kinematics of the exponential coordinates is employed. The control scheme is designed to asymptotically reduce the relative configuration and relative velocities autonomously from almost any given initial state, except those that differ in orientation by a π radian rotation from the desired states at the initial time. Note that the exponential coordinates for the relative attitude (orientation) are not uniquely defined in this case [3,7,18]. Since the set of such initial states is an embedded lower-dimensional subspace of the state space, this tracking control scheme is therefore *almost global* in its convergence over the state space [23]. This makes it possible to apply this feedback tracking scheme for spacecraft hovering control over a large range of relative motion between the asteroid and spacecraft.

This formulation, based on geometric mechanics, naturally considers the coupled translational (orbital) and attitude motion of spacecraft leading to six-degrees-of-freedom motion, without having to design separate controllers for the orbital and attitude motions. The attitude motion is dynamically coupled to the translational motion, moreover, translational control is dependent on the spacecraft attitude for thrust direction. The thrust direction error leads to the need for a coupled attitude and translational control strategy. Moreover, sensor measurements made by the spacecraft are usually available in the spacecraft's body-fixed frame. The coupled attitude and orbital controller design presented here can maximize the control efficiency of the spacecraft, by naturally considering the coupled translational and attitude dynamics. It is assumed that the mass and inertia properties remain constant during operation of actuators.

The novelty of this study is summarized as follows. An asymptotic tracking control for autonomous body-fixed hovering of a rigid spacecraft over an asteroid is proposed in the framework of geometric mechanics without having to design separate controllers for the orbital and attitude maneuvers. The asymptotic tracking control is a continuous-time feedback tracking control which can perform coupled translational and rotational maneuver over an asteroid. Thus, the spacecraft in this study is considered as a rigid spacecraft whose motion is described by both position and attitude instead of a point mass spacecraft. Then, the states are updated by the feedback control law such that the relative configuration and velocities can converge to zero without using explicit reference states or the help of explicit guidance unlike conventional control schemes [14,20,32,33]. Thus, coupled translational and rotational maneuvers are performed autonomously to achieve the desired relative configuration and velocities from any given configuration and velocity conditions. In addition, the feedback control law corrects and updates the implicit reference states expressed in the exponential coordinates leading to the desired configuration. This ensures that the spacecraft can perform body-fixed hovering maneuver comprised of approach and stationkeeping phases of the maneuver autonomously with one single control law. Thus, the feedback control law corrects and updates the implicit reference