Aerospace Science and Technology 34 (2014) 1–11

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

State-space representation of the unsteady aerodynamics of flapping flight

Haithem E. Taha [a,*,1], Muhammad R. Hajj [a,2], Philip S. Beran [b,3]

[a] Virginia Polytechnic Institute and State University, Blacksburg, VA, 24061, United States
[b] U.S. Air Force Research Laboratory, Wright-Patterson Air Force Base, Ohio 45433, United States

ARTICLE INFO

Article history:
Received 14 October 2013
Received in revised form 23 January 2014
Accepted 28 January 2014
Available online 11 February 2014

Keywords:
Flapping flight
Leading Edge Vortex
Aspect ratio effects
Unsteady aerodynamics
Indicial response
Duhamel's principle

ABSTRACT

A state-space formulation for the aerodynamics of flapping flight is presented. The Duhamel's principle, applied in linear unsteady flows, is extended to non-conventional lift curves to capture the LEV contribution. The aspect ratio effects on the empirical formulae used to predict the static lift due to a stabilized Leading Edge Vortex (LEV) are provided. The unsteady lift due to arbitrary wing motion is generated using the static lift curve. Then, state-space representation for the unsteady lift is derived. The proposed model is validated through a comparison with direct numerical simulations of Navier–Stokes on hovering insects. A comparison with quasi-steady models that capture the LEV contribution is also performed to assess the role of unsteadiness. Similarly, a comparison with classical unsteady approaches is presented to assess the LEV dominance. Finally, a reduced-order model that is more suitable for flight dynamics and control analyses is derived from the full model.

1. Introduction

The aerodynamics of flapping flight have been the focus of research investigations for almost a century. The early studies were concerned with birds and insect flights and mainly carried out by biologists, such as Demoll [12,13]. More recently, there has been a significant interest in the modeling and simulation of flapping flights for design of micro-air-vehicles (MAVs). Flapping flight of MAVs/insects generates an unsteady nonlinear flow field that exploits non-conventional mechanisms to enhance the aerodynamic loads. Almost all of the early trials of explaining insect flight have invoked non-conventional high-lift mechanisms. Ellington et al. [20] explained how insects exploit the Leading Edge Vortex (LEV) as a high-lift mechanism, which is also known to be critical for lift generation of highly swept and delta wings aircraft. The LEV augments the bound vortex on the wing and, as such, the lift increases. This phenomenon is similar to the one observed in dynamic stall whereby the wing undergoes a rapid variation in the angle of attack. Yet, in contrast to dynamic stall, the LEV formed in insect flight has stable characteristics. This stability is attributed to an outward spanwise flow that convects the LEV towards the wing tip [20,54,53,61]. In the case of highly swept and delta wings, this spanwise flow is generated by the free-stream component parallel to the highly swept leading edge. In insect flight, similar to helicopters and propellers, the rotational motion creates a spanwise velocity gradient which, in turn, creates a pressure gradient that generates the spanwise flow.

Although the LEV is known to be the dominant contribution in insect flight, Dickinson et al. [15] indicated two other high-lift mechanisms, namely the rotational lift and wake-capture effects. The rotational lift is mainly due to the wing rotation at the end of each half stroke to adjust the angle of attack for the next half stroke. This rotational velocity of the wing creates a circulation that induces additional aerodynamic lift. On the other hand, Dickinson et al. observed peaks in the generated lift at the beginning of half strokes, when forward speed of the wing is almost zero. These peaks could not be explained by the previous two mechanisms. Dickinson et al. related these peaks to the lingering wake created during the previous half stroke. In addition to the non-conventional high-lift mechanisms discussed above, the role of unsteady aerodynamics in flapping flight is also quite significant. Other flow aspects that affect the aerodynamic loads include non-circulatory and viscous friction contributions. Unfortunately, it is very difficult to formulate a model for the aerodynamic forces that accurately captures all these phenomena without an expensive computational burden.

Over the two past decades, significant advancements have been made towards the understanding and modeling of the

* Corresponding author.
E-mail addresses: hezzat@vt.edu (H.E. Taha), mhajj@vt.edu (M.R. Hajj), philip.beran@wpafb.af.mil (P.S. Beran).

[1] Post-Doctoral Research Fellow, Engineering Science and Mechanics, Norris Hall.
[2] Professor, Engineering Science and Mechanics, Norris Hall.
[3] Principal Research Aerospace Engineer, AFRL/RBSD.

http://dx.doi.org/10.1016/j.ast.2014.01.011
1270-9638/

Nomenclature

AR	Aspect ratio	S	Area of one wing
c	Chord length	s, σ	Non-dimensional time variables
$\bar{c}$	Mean chord length	t, τ	Time variables
C_D	Drag coefficient	T, f	Flapping period and frequency
C_L	Lift coefficient	U	Air speed
$C_{L,s}$	Static lift coefficient	$W(s)$	Wagner function
$\overline{C_L}$	Average lift coefficient	w	Wing normal velocity
C_{L_α}	Lift curve slope of the three-dimensional wing	ω	Flapping angular frequency
$C(k)$	Theodorsen function	$\widehat{x}_0$	Normalized position of the pitch axis
$D(p)$	Theodorsen function in the Laplace-domain	α	Angle of attack
k	Reduced frequency	η	Pitching angle
ℓ	Lift per unit span	φ	Back and forth flapping angle
ℓ_s	Static lift per unit span	ρ	Air density
p	Laplace variable	ϑ	Plunging angle
r	Distance along the wing span	LEV	Leading Edge Vortex
R	Wing radius (length)	$UVLM$	The unsteady vortex lattice method
Re	Reynolds number		

Table 1
The aerodynamic models in the literature that could be applied to hovering MAVs/insects and the physical aspects associated with the aerodynamics of flapping flight that each of the listed models captures along with the degrees-of-freedom associated with that model. UVLM refers to the unsteady vortex lattice method.

	Dickinson et al. [15]	Berman and Wang [8]	Peters et al. [39,34]	UVLM	Ansari et al. [5,6]	Proposed model
No. degrees-of-freedom	low	low	low	high	high	low
LEV contribution	✓	✓	×	×	✓	✓
Unsteadiness	×	×	✓	✓	✓	✓
Rotational lift	✓	✓	✓	✓	✓	✓
Added mass	✓	✓	✓	✓	✓	✓
Wake capture (hovering)	×	×	×	✓	✓	×
Viscous friction	×	✓	×	×	×	×

aerodynamics of flapping flight. For detailed reviews, the reader is referred to Mueller [31], Shyy et al. [46], Sane [42], Wang [57], and Ansari et al. [4]. Taha et al. [50] provided a review for the aerodynamic models specifically used in flight dynamics and control analyses. Table 1 lists the aerodynamic models that are available in the literature to be applied to hovering MAVs/insects. Also, we list the physical aspects associated with the aerodynamics of flapping flight that each of the listed models captures along with the degrees-of-freedom associated with that model. The first two models have algebraic forms and the third one comprises finite-state ordinary differential equations. The next two models involve simulation of the vortex kinematics at many locations on the airfoil surface and in its wake. Clearly, the first three models have a lower computational cost than the next two and hence are better-suited for flight dynamics and control analysis. Wang and Eldredge [59] proposed a remedy for the high computational cost associated with Ansari's model. Instead of shedding constant-strength point vortices at each time step from both leading and trailing edges, they shed variable-strength point vortices at larger time lapses. According to their shedding criterion, the strengths of the point vortices are determined at each time step by satisfying the Kutta condition at the edge it has shed from until an extremum value is reached. Then, the strength of this point vortex is kept constant and a new vortex is shed at this instant. This formulation greatly reduces the number of degrees-of-freedom. However, there is still a need to develop an unsteady model in a compact form that is suitable for aeroelasticity, flight dynamics, and control synthesis. Brunton and Rowley [10] considered Theodorsen's model of the lift frequency response [51] and modified its coefficients to be suitable for low Reynolds number regime. So, their final model has the same form as Theodorsen's but with different coefficients (different amplitudes). This cannot account for the LEV effect, which is our main concern in this work. Thus, it is concluded from Table 1 and the above discussion that there is a need for an aerodynamic model that captures the dominant LEV contribution along with the prominent unsteadiness with a feasible number of degrees-of-freedom so that it could be used in flight dynamics analysis, control synthesis, optimization, and sensitivity analysis.

More generally, Fig. 1 presents a taxonomy of the flapping flight regimes. For forward flights with a low reduced frequency k, typically $k < 0.1$, the quasi-steady aerodynamics is applicable. For forward flight with a relatively high k with local angles of attack up to 25°, a number of aerodynamic theories can be applied to capture the unsteadiness with a good accuracy either for two-dimensional or three-dimensional wings, e.g., Theodorsen [51], Shwarz and Sohngen (see [9]), Peters et al. [39,34,36,35,37,38], Jones [24–26], and Reissner [41]. In addition, methodologies such as the unsteady lifting line theory, the unsteady vortex lattice method, and the unsteady doublet lattice method can also be used to capture the unsteady effects on three-dimensional wings. On the other hand, for hovering with very high flapping frequency ω relative to the body natural frequency ω_n, it is generally assumed that there is no coupling between the periodic aerodynamic forces and the body natural modes [50]. As such, the body feels only the average forces, which might be predicted by the quasi-steady models that capture the dominant effect (LEV), for example, Dickinson et al. [15], Pesavento and Wang [33], and Andersen et al. [2,3]. For the middle regimes in Fig. 1, there is no aerodynamic model that could cover this gap with a feasible computational burden. The main characteristics of this regime are the LEV contribution, the prominent unsteadiness, and the coupling between the periodic aerodynamic forces and the body modes. The objective of this work is to develop a physics-based model in the form of ordinary-differential equations that describe the lift buildup during the flapping cycle, including the effect of the LEV on the aerodynamic loads. This model can provide better assessment of the flapping