Aerospace Science and Technology 32 (2014) 66–83

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Elastic/plastic buckling of isotropic thin plates subjected to uniform and linearly varying in-plane loading using incremental and deformation theories

M. Kadkhodayan *, M. Maarefdoust

Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad 91775-1111, Iran

ARTICLE INFO

Article history:
Received 16 July 2013
Received in revised form 8 November 2013
Accepted 7 December 2013
Available online 12 December 2013

Keywords:
Thin plate
GDQ method
Elastic/plastic buckling
Linearly varying loading

ABSTRACT

The present study is concerned with the elastic/plastic buckling of thin rectangular plates under various loads and boundary conditions. The in-plane loads are placed uniformly and linearly varying in the uniaxial compression and biaxial compression/tension. The equilibrium and stability equations are derived and analyses are carried out based on two theories of plasticity, i.e. deformation theory (DT) and incremental theory (IT). The elastic/plastic behavior of plates is described by the Ramberg–Osgood model. Generalized Differential Quadrature (GDQ) discretization technique is used to solve the buckling of plate equation. To examine accuracy of the present formulation and procedure, several convergence and comparison studies are investigated and new results are presented. The differences between the IT and DT results increase by increasing loading parameter in linearly varying in-plane loading. Some new consequences are achieved regarding the validation range of two theories. Furthermore, effects of aspect, thickness to length and loading ratios, boundary condition, type of plasticity theory and linearly varying in-plane loading on the buckling coefficient are discussed. Contour plots of buckling mode shapes for various loading parameters are also illustrated.

1. Introduction

The elastic/plastic buckling of plate, widely used in aerospace, mechanical, civil and marine engineering structures are concerned by many of the engineers. Moreover, plates are extensively used in structures such as aircraft wings and bridges. Then it is important to know buckling capacities of the structures in order to avoid premature failure. For large amounts of loading the buckling phenomena may occur in the plastic range. This phenomenon may be likely to occur in the cases of plates whose materials posses a low proportional limit when compared to the nominal yield stress, for example aluminum alloy and stainless steel. Researchers have given considerable attention to the buckling of plates issue numerically and analytically in both elastic and plastic buckling modes. They have investigated the plastic buckling behavior of plates subjected to uniaxial and biaxial compression loadings using two theories of strain-hardening plasticity, incremental and deformation theories. It would be useful for the design profession if a simple computing algorithm is available, that is rapidly adaptable to specific problems in determining the plastic bifurcation buckling loads.

Ilyushin [15], Handelman and Prager [13], Stowell [26] and Pride and Heimerl [18] carried out the plastic buckling analysis with incremental and deformation theories, respectively. They showed that the results attained by DT are close to the experimental results. Tugcu [28] illustrated that the analysis based on IT is more sensitive than DT with the test parameters. Geier and Singh [11] presented a simple analytical solution for computing bifurcation buckling loads of thin and moderately thick orthotropic cylindrical shells and panels subjected to axial compression and normal pressure. The analysis was based on the governing nonlinear equations. Durban [8] found out that the IT could predict more buckling load in comparison with DT, and that the experimental data had more congruence with DT. Of course, there are some cases where the critical stresses obtained from two theories are nearly equal. A typical example is furnished by axially symmetric buckling of axially compressed circular cylindrical shells. Durban and Zuckerman [9] carried out the analysis of rectangular plates under uniaxial loading for several various modes with the separation of variables solution. However, the limited boundary conditions consisting of clamped and simply supported have limited the obtained data in that research. If all boundary conditions are clamped, then it would

* Corresponding author. Tel.: +98 9153111869; fax: +98 5118763304.
E-mail address: kadkhoda@um.ac.ir (M. Kadkhodayan).

1270-9638/$ – see front matter
http://dx.doi.org/10.1016/j.ast.2013.12.003

Nomenclature

a, b	Plate lengths in x- and y-directions, respectively
$C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}, C_{ij}^{(4)}$	The weighting coefficients of the first, second, third and fourth-orders
n, k	Ramberg–Osgood parameters
D	Flexural rigidity of plate [$\equiv Eh^3/12(1-\upsilon^2)$]
E	Young's modulus of elasticity
G	Effective shear modulus
h	Thickness of plates
h/a	Thickness to length ratio
K	Buckling coefficient [$\equiv Pa^2h/\pi^2D$]
N_x, N_y	Number of grid point in the x- and y-directions, respectively
P_0	The maximum intensity of compressive force at the edge of plate in Eq. (29)
P_x	In-plane compressive forces per unit length of the plate in the x direction in Eq. (29)
S_{ij}	Stress deviator tensor
$S(E_s)$	Secant modulus
$T(E_t)$	Tangent modulus
U	Strain energy
V	Potential energy
w	Transverse deflection of the plate
X, Y, W	Non-dimensional parameters

Greek symbols

$\alpha, \beta, \gamma, \chi, \mu, \delta$	Parameters used in stress–strain relations
η	Loading parameter in Eq. (29)
ε_e	Total effective strain
ε	Total strain
λ	Aspect ratio [$\equiv a/b$]
υ	Poisson's ratio
σ_e	Effective stress
ξ	Loading ratio

be difficult to solve the problem and other numerical methods have to be used. In the present study, it is aimed to solve all these restrictions through generalized differential quadrature method.

Betten and Shin [6] showed that if the plate is slender, the buckling is elastic. However, if the plate is sturdy, it buckles in the plastic range and the instantaneous moduli in the constitutive equations depend on the external loading. Wang et al. [31,32,29] investigated the elastic–plastic buckling of thin and thick plates based on deformation and incremental theories by use of separation of variables and Ritz method. They came to the conclusion that the DT predicts less buckling stress factor, and as the thickness and Ramberg–Osgood constant increase, the differences between two theories increase. Smith et al. [25] studied the inelastic buckling of steel plates based on classical theory under different loading conditions by using Rayleigh–Ritz method. El-Sawy et al. [10] have employed the finite element method (FEM) to determine the elasto–plastic buckling stress of uniaxially loaded square and rectangular plates with circular cutouts. They showed that the critical buckling stress for perforated plates always decreases as the plate slenderness ratio increases and that this decrease becomes steeper for larger values of plate slenderness ratios, especially for small hole sizes where the failure changes from elasto–plastic into pure elastic. Grognec and Van [12] used the 3D plastic bifurcation theory assuming the incremental theory of plasticity with the von Mises yield criterion and a linear isotropic hardening. Wang et al. [30,34] studied the elastic/plastic buckling of thick and thin plates by differential quadrature method and confirmed the results of Refs. [9,31]. Aydin Komur [2] studied the effect of plate aspect ratio, elliptical hole size, angle and location and slenderness ratio on buckling behavior. He found that as the plate slenderness ratio increases, the critical buckling stress decreases for all perforated plates.

More recently, Robert et al. [20] compared the incremental and deformation theories and flow rules in simulating the sheet-metal forming processes. It can be concluded that the major advantage of the new approach was the time benefit when the material non-linearities were dominant. Weißgraeber et al. [33] studied the buckling behavior of an orthotropic plate with elastic clamping and edge reinforcement under uniform compressive load. Rahimi et al. [19] analyzed the buckling behavior of thin-walled cylindrical shells under axial force by finite element analysis method. They showed that stiffening the shells increased the buckling load while decreased the buckling load to weight ratio of an unstiffened shell.

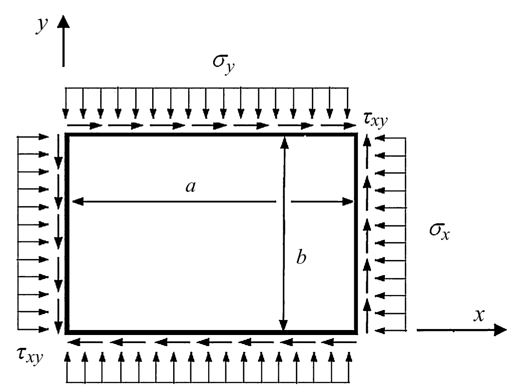

Fig. 1. Geometry and loading conditions of a rectangular plate.

Ruocco and Mallardo [21] applied a model to predict the buckling behavior of thin, orthotropic, stiffened plates and shells subjected to axial compression. The equilibrium equations have been solved applying the Kantorovich method. They showed that the Von Karman model could sensibly overestimate the critical load. Chakrabarty [7] pointed out the difference between bifurcation and stability and presented the equations used for the analyses. However, the point that the differences between the obtained results from two theories of deformation and incremental are observable is still a paradox.

In this work, details of elastic/plastic buckling of thin rectangular plates using incremental and deformation theories of plasticity are introduced first. An important criterion for sizing and certification of aircraft fuselages is the local and global buckling behavior. Therefore it is necessary to know the buckling behavior as accurately as possible. The uniform and non-uniform in-plane axial and biaxial tension/compression loadings with various boundary conditions are considered for the first time. The material properties described by the stress–strain relationship proposed by the Ramberg–Osgood stress–strain model. The GDQ method as an efficient numerical tool is employed to establish an eigenvalue problem and to calculate the plate buckling coefficients. The validation of the GDQ solutions by comparison with corresponding results for a typical aerospace aluminum alloy (AL 7075-T6) material is described. The numerical results are presented to show the effect of aspect, thickness to length and loading ratios, boundary condition, type of plasticity theory and linearly varying in-plane loading on the buckling coefficient of plates.